Name: ______________________________

Student ID number: ______________________________

Guidelines for the test:

- Put your name or student ID number on every page.
- There are 12 problems: 6 problems in Part I (7 points each) and 6 problems in Part II . If you got more than 100 points, only 100 points were counted. (超過100分以100分計算)
- The exam is closed book; calculators are not allowed.
- There is no partial credit for problems in the Part I (multiple-choice (選擇) and fill-in (填充) problems).
- For problems in the Part II (problem-solving (計算題) problems), please show all work, unless instructed otherwise. Partial credit will be given only for work shown. Write as legibly as possible - correct answers may have points taken off, if they're illegible.
- Mark the final answer.

Part I: (7 points for each problem)

Multiple Choice - Single Answer (選擇題- 單選題).

(1) Given $f(x,y) = x^3 - 3x + y^2 + 9$, which of the following is correct?

A) (-1,1) is a saddle point
B) (2,0) is a local minimum
C) (1,0) is a local minimum
D) all of the above

(2) R is a region bounded by $y = 4x + 2$, $y = 4x + 5$, $y = 3 - 2x$ and $y = 1 - 2x$, Given the transformation $u = y - 4x, v = 2x + y$, which of the following is equivalent to $\int\int_R x^2\, dA$?

A) $\int_2^5 \int_1^3 x^2\, dudv$,
B) $\int_2^5 \int_1^3 (\frac{(v-u)}{6})^2\, dudv$,
C) $\int_2^5 \int_1^3 (\frac{(v-u)}{6})^2(-\frac{1}{3})\, dudv$
D) $\int_2^5 \int_1^3 (\frac{(v-u)}{6})^2 \frac{1}{3}\, dudv$
E) None of the above. Ans=?

Fill-In Problems(填充)

(3) Find an equation of the plane containing the point $(3,2,1)$ with normal vector $< 4, 5, 6 >$.

$Ans =$____ .

(4) Given that $f(x,y) = x^3 - 3x + y^2 + 9$, compute the directional derivative of f at $(1,2)$ in the direction of the vector $< \frac{\sqrt{3}}{2}, \frac{1}{2} >$

$Ans =$____ .

(5) Suppose that $g(t) = f(x(t), y(t))$, where f is a differentiable function of x and y and where $x = x(t)$ and $y = y(t)$ both have first-order derivatives. Given that

$x'(1) = 2$, $y'(1) = 3$ $x(1) = 4$, $y(1) = 5$,
$f(4,5) = 6$, $f_x(4,5) = 7$, $f_y(4,5) = 8$,

$g'(1) =$____ .

(6) Find the equation of the tangent plane to the surface at the given point.

$$z = x^2 - y^2 + 1 \quad \text{at } (1,2,-2)$$

$Ans =$____ .

Part II:

Problem-Solving Problems (計算題 **Show all work**)

(7) Given that $\mathbf{r}(t) =< t^2 - t, e^{3t}, 0 >$, calculate

- $\lim_{t\to 0} \mathbf{r}(t) =?$
- $\frac{d}{dt}\mathbf{r}(t) =?$
- $\int \mathbf{r}(t)\, dt =?$
- $\int_0^1 \mathbf{r}(t)\, dt =?$

(8) Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{12xy^2}{x^2 + y^4}$$

(9) Given $f(x,y) = x^3 - e^{xy} + y^3$, find the partial derivatives, f_x, f_y, f_{xy} and f_{xx}.

(10) For a differentiable function $g(u,v) = f(x(u,v), y(u,v))$ with $f(x,y) = 10x^{1/4}y^{3/4}$, $x(u,v) = u + v$ and $y(u,v) = u - v$, use the **chain rule** to find $g_u(u,v)$ and $g_v(u,v)$.

(11) Find the maximum and minimum values of the function $f(x, y) = x^2 + y^2 + 6y$ subject to the constraint $x^2 + 4y^2 \leq 4$

(12) Evaluate the iterated integral by first changing the order of integration.

$$\int_0^2 \int_{y/2}^1 e^{x^2} \, dx \, dy$$