Calculus II Final Thursday, June 15, 2006

No calculator is allowed. No credit will be given for an answer without reasoning
Note that the total is more than 100, but you can only get 100 if you score higher

1. (6 points) Show absolute, conditional convergence or divergence for the following series.

$$\sum_{k=1}^{\infty}(-1)^k \frac{k^2}{(2k)!}.$$

2. (14 points total)

   (a) (2 points) Find the Taylor series of $g(t) = e^t$ at $t = 0$.

   (b) (4 points) Using your answer to (a), find the Taylor series of $f(t) = te^t$ at $t = 0$.

   (c) (4 points) Using your answer to (b), find the Taylor series of

   $$F(x) = \int_0^x te^t\,dt$$

   at $x = 0$.

   (d) (4 points) From your solution to (c), show that

   $$\sum_{k=0}^{\infty} \frac{1}{(k+2)(k!)} = 1.$$

3. Given the position function of a particle

$$\mathbf{r}(s) =< \frac{(1+s)^{3/2}}{3}, \frac{(1-s)^{3/2}}{3}, \frac{s}{\sqrt{2}} >,$$

   (a) (2 points) find the velocity, $\mathbf{v}(t)$,

   (b) (3 points) show that the particle has unit speed,

   (c) (5 points) find the unit tangent vector $\mathbf{T}(t)$ and the principal unit normal vector $\mathbf{N}(t)$

4. (8 points) For a differentiable function $g(u, v) = f(x(u, v), y(u, v))$ with $x(u, v) = u\cos v$ and $y(u, v) = u\sin v$ and where f_{xy} and f_{yx} are continuous, use the **chain rule** to find f_u and f_v.

$$f(x, y) = 4x^3y^2$$

5. (8 pts) Find local maximum and minimum values and saddle points of the function

$$f(x, y) = 2x^2 + y^2 + 2x^2y + 9$$

6. (8 pts) **Use Lagrange Multipliers** to find the maximum and minimum values of the function $f(x, y) = e^{xy}$ subject to the constraint $x^2 + y^2 = 8$

7. (8 pts) Evaluate the iterated integral by first changing the order of integration.

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{2}} 3e^{x^2}\, dx\, dy$$

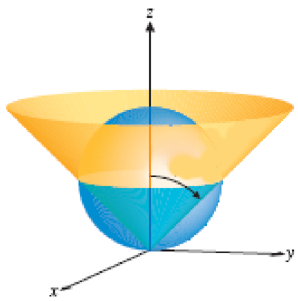
8. (8 pts) Evaluate the iterated integral by converting to polar coordinates.

$$\int_{-2}^{0} \int_0^{\sqrt{4-x^2}} \sin\left(x^2+y^2\right) dy\, dx$$

9. (12 points) Q is the solid inside the sphere $x^2+y^2+z^2=4z$ and inside the cone $z^2=(x^2+y^2)/3$.

   (a) Set up but **Do Not Evaluate** the triple iterated integral for the volume of Q in Cylindrical coordiantes.

   (b) Set up but **Do Not Evaluate** the triple iterated integral for the volume of Q in Spherical coordiantes.



10. (8 points) Evaluate the integral $\displaystyle\int\int_R y+3x\, dA$, where R is the region bounded by $y=3-3x$, $y=1-3x$, $y=x-3$ and $y=x-1$, by changing variables.

11. (8 points) Use the Divergence Theorem compute $\displaystyle\int\int_{\partial Q} \vec{F}\cdot\vec{n}\, dS$,

    wher Q is bounded by $z=\sqrt{4-x^2-y^2}$ and $z=0$, $\vec{F}=<z^3, x^2z, xy^2>$

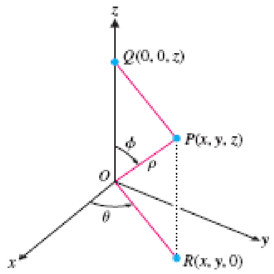
12. Use Stokes' Theorem to evaluate

   (a) (6 points) $\int_C \vec{F}\cdot d\vec{r}$.
   where C is the circle $x^2+y^2=9$ on the xy-plane, oriented so that it is traversed counterclockwise when view from the positive z-axis and $\vec{F}=<x^2, y^4-x, z^2\sin z+x>$

   (b) (4 points) $\int\int_S(\nabla\times\vec{F})\cdot\vec{n}\, dS$ where S is $z=9-x^2-y^2$ above the xy-plane with $\vec{n}$ upward and $\vec{F}=<x^2, y^4-x, z^2\sin z+x>$.

- Double-Angle
  $\sin 2\theta = 2\sin\theta\cos\theta$ $\qquad$ $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

- Spherical Coordinates:



- Curl and Divergence:

$$\begin{aligned} curl\vec{F} = & \quad \nabla \times \vec{F}, \\ div\vec{F} = & \quad \nabla \cdot \vec{F}, \\ \nabla = & \quad < \partial_x, \partial_y, \partial_z > \end{aligned}$$

- Divergence Theorem:
  Suppose that $Q \subset R^3$ is bounded by the closed surface ∂Q and that $\vec{n}(x, y, z)$ denotes the exterior unit normal vector to ∂Q. Then, if the components of $\vec{F}(x, y, z)$ have continuous first partial derivatives in Q, we have

$$\int\int_{\partial Q} \vec{F} \cdot \vec{n}\, ds = \int\int\int_{Q} \nabla \cdot \vec{F}(x, y, z)\, dV$$

- Surface Area Element (if the surface is given by $z = f(x, y)$:

$$ds = \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

- Surface Area Element (if the surface is given by $\vec{r}(u, v)$):

$$ds = ||\vec{r}_u \times \vec{r}_v||dA$$

- Stokes Theorem:
  Suppose that **S** is an oriented, piecewise-smooth surface, bounded by the simple closed, piecewise-smooth boundary curve ∂**S** having positive orientation. Let $\vec{F}(x, y, z)$ be a vector field whose components have continuous first partial derivatives in some open region containing **S**. Then,

$$\int_{\partial S} \vec{F} \cdot d\vec{r} = \int\int_{S} (\nabla \times \vec{F}) \cdot \vec{n}\, dS$$

- For $\vec{r}(t) =< x(t), y(t), z(t) >$,

$$d\vec{r} =< dx, dy, dz >=< x', y', z' > dt$$