No credit will be given for an answer without reasoning. Recall that  
\[
\cosh x = \frac{1}{2}(e^x + e^{-x}); \sinh x = \frac{1}{2}(e^x - e^{-x}).
\]

1. [10%] Consider the curve \(\alpha(t) = (2t, t^2, t^3/3)\).
   (i) Let \(T\) be the unit tangent vector of \(\alpha(t)\). Find \(\lim_{t\to\infty} T\).
   (ii) Compute the curvature \(\kappa\).

2. [10%]
   (i) Let \(V, W\) be vector fields on \(\mathbb{R}^3\) and \(f\) be a differentiable function on \(\mathbb{R}^3\). Show that  
   \[
   \nabla_V (fW) = V[f]W + f\nabla_V W.
   \]
   (ii) Let \(U_1 = (1, 0, 0), U_2 = (0, 1, 0)\) and \(U_3 = (0, 0, 1)\), \(V = -yU_1 + xU_3\) and \(W = \cos xU_1 + \sin xU_2\). Find \(\nabla_V W\) (in terms of \(U_1, U_2, U_3\)).

3. [10%] Let \(M, N\) be two surfaces in \(\mathbb{R}^3\) and let \(F: M \to N\) be a mapping of surfaces. Show that the tangent map \(F^*: T_p(M) \to T_{F(p)}(N)\) is a linear transformation.

4. [10%] Let \(M\) be the catenoid:  
   \[
   \mathbf{x}(u, v) = (u, \cosh u \cos v, \cosh u \sin v).
   \]
   Define  
   \[
   E_1 = \frac{\mathbf{x}_u}{||\mathbf{x}_u||}, \quad E_2 = \frac{\mathbf{x}_v}{||\mathbf{x}_v||}, \quad E_3 = E_1 \times E_2.
   \]
   (i) Check that \(E_1, E_2, E_3\) form a adapted frame field on \(M\).
   (ii) Find the dual form \(\theta_1\) of \(E_1\).

5. [10%] Let \(M\) as in Problem 4.
   (i) Describe the image of Gauss map of \(M\).
   (ii) Find the Gaussian curvature \(K\) of \(M\).

6. [10%] Let \(N\) be the helicoid:  
   \[
   \mathbf{y}(s, t) = (s \cos t, s \sin t, t).
   \]
   Show that \(N\) is a minimal surface.

7. [10%] Let \(M\) be as in Problem 4 and \(N\) be as in Problem 6. Define the map \(F: N \to M\) by  
   \[
   F(\mathbf{y}(s, t)) = \mathbf{x}(\sinh u, v).
   \]
   Show that \(F\) is a local isometry from \(N\) to \(M\).

8. [10%] Let \(M\) be the cylinder \(x^2 + y^2 = 1\). Compute the distance \(\rho(p_1, p_2)\) on \(M\) where \(p_1 = (1, 0, 0)\) and \(p_2 = (-1, 0, 1)\) are two points of \(M\).
9. [10%]
(i) Let $M$ be the sphere: $x^2 + y^2 + z^2 = r^2$. Consider the points $A = (r, 0, 0)$, $B = (0, r, 0)$ and $C = (0, 0, r)$ on the sphere and connect any two of them by great circles. Use Gauss-Bonnet theorem to show that $\angle A + \angle B + \angle C = 3\pi/2$.
(ii) Let $N$ be the cylinder $x^2 + y^2 = 1$. Let $P$ be the plane $y - z = 0$ and $\alpha$ be the intersection of $N$ and $P$. Let $k_g$ be the geodesic curvature of $\alpha$ considered as a curve on $N$. Find $\int_{\alpha} k_g \, ds$.

10. [10%]
(i) Let $\ell$ be the curve $\alpha(t) = (t, t^2, 0)$. Find a surface $M \subset \mathbb{R}^3$ such that $\ell$ is a geodesic on $M$.
(ii) Does there exist a compact minimal surface? Why or Why not?