Optimal Control of Stochastic Inventory System with Multiple Types of Reverse Flows

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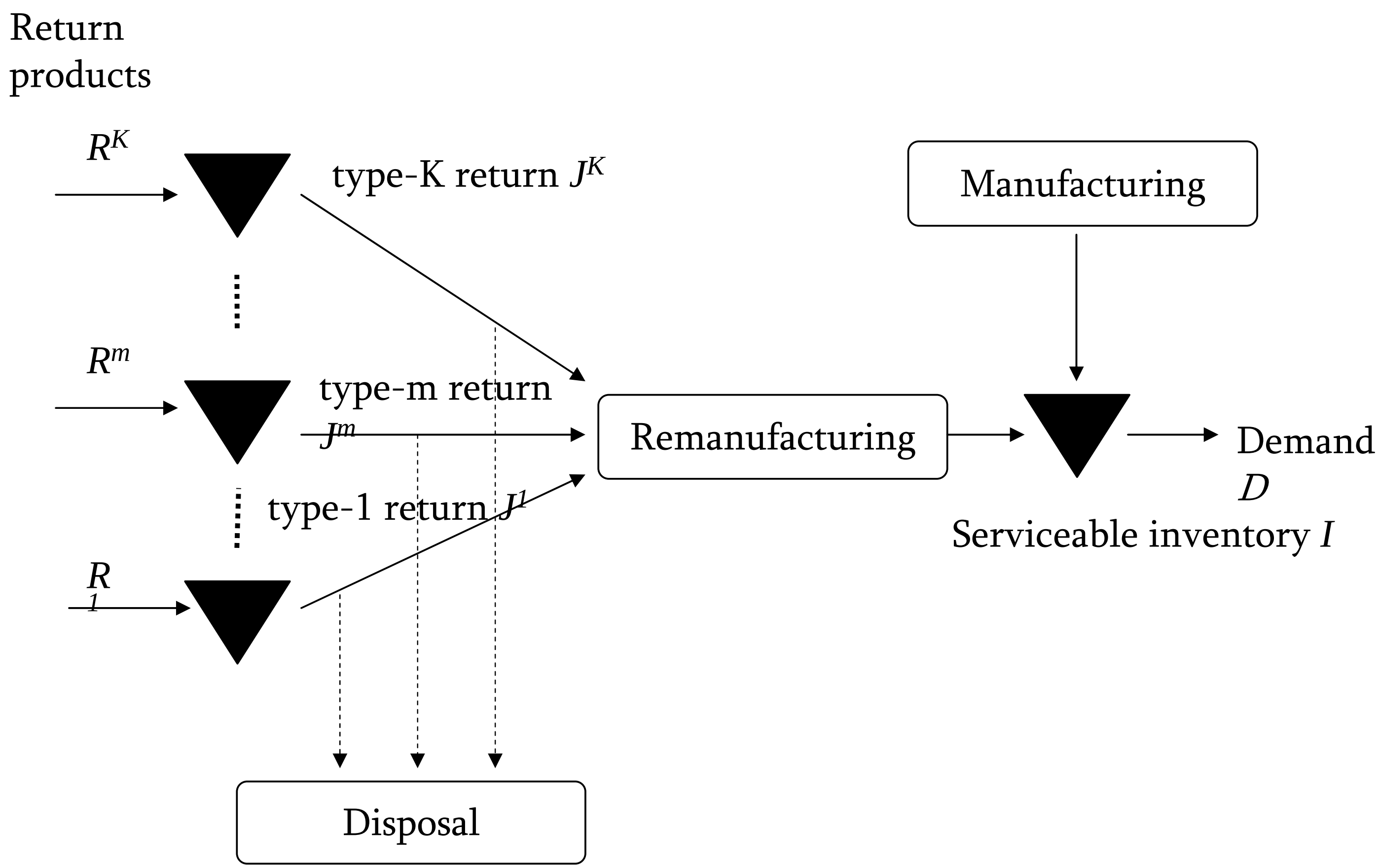
Joint work with S. Zhou and Z. Tao

The Problem

- Single period problem – Newsvendor problem
- Multiple-period problem
- Optimal strategy – Base-stock policy
- Multiple types of returns – Motivating examples

The Problem



Return products
R^K
type-K return J^K
Manufacturing
R^m
type-m return J^m
Remanufacturing
Demand D
type-1 return J^1
R_1
Serviceable inventory I
Disposal

Some related literature

- Simpson (1978)
- Inderfurth (1997)
- Decroix (2006)
- Decroix and Zipkin (2005)
- All these papers consider a single type of returned products.

Other related and review articles

- Heyman (1977)
- van der Laan, et al. (1999)
- van der Laan and Teunter (2005)
- Fleischmann et al. (1997)
- Guide and Srivastava (1997)

Model Details

- Periodic review system, periods 1 to N.
- K types of returned products.
- Disposal may or may not be allowed.
- Manufacturing and remanufacturing times are equal, and are assumed, without loss of generality, to be 0.
- The demand for serviceable product over the periods are $D_1, D_2, \ldots, D_N$.

Cost Structure

- Production cost (or ordering cost) for serviceable product, p.
- Repair cost for type j return is r_j, where $p \geq r_j$, $j = 1, \ldots, K$.
- Stocking (holding) cost for type i return is s_i, $i = 1, \ldots, K$.
- WLOG, assume

$$(1-\alpha)r_1 - s_1 \leq \cdots \leq (1-\alpha)r_K - s_K.$$

Cost Structure (Cont'd)

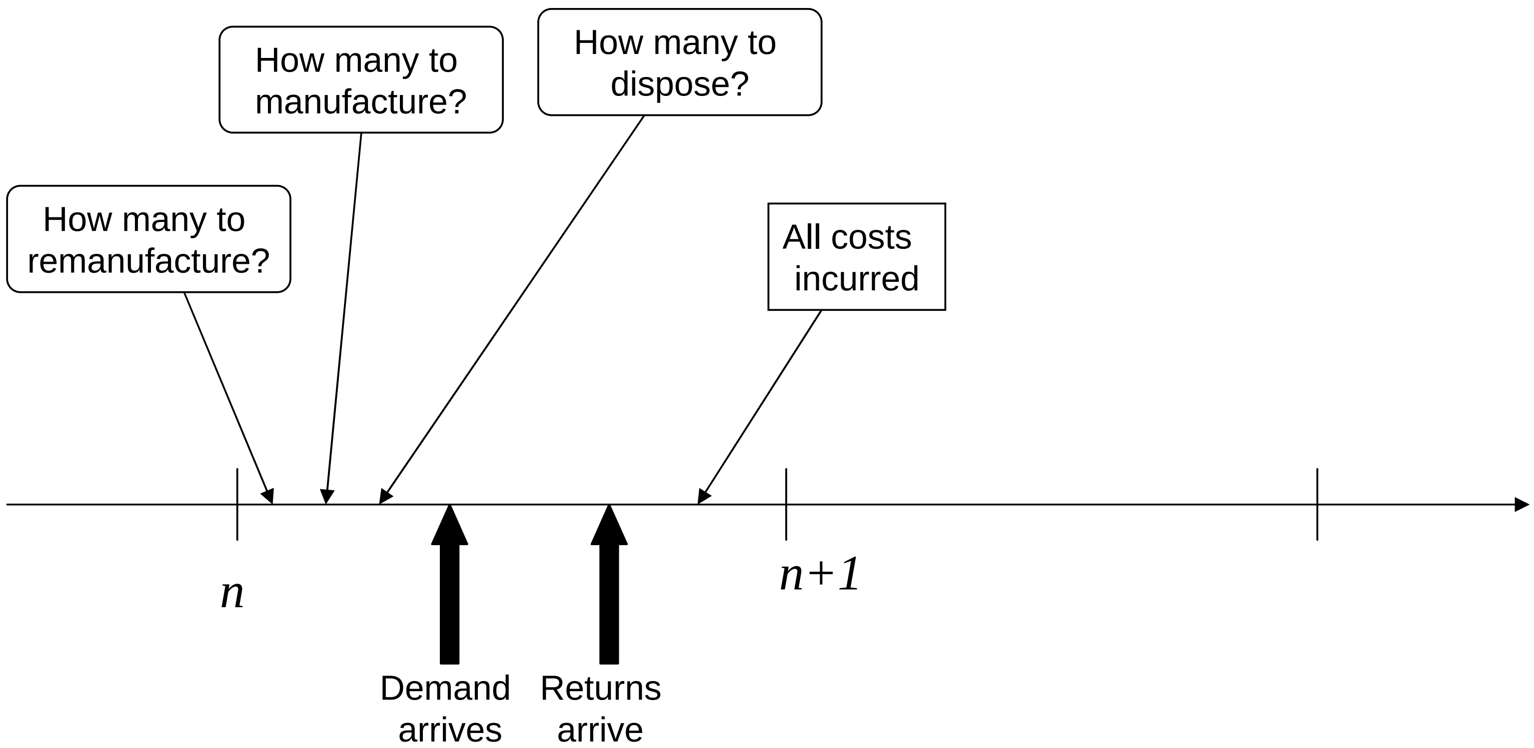
- There is holding cost for serviceable product.
- Consider backlog model (lost-sales model can be similarly studied)– shortage cost for backlog
- Holding cost for serviceable product and shortage cost for backlog is a general convex function: Expected one-period cost $G(x)$, i.e.,

$$G(x) = hE[\max\{x - D, 0\}] + bE[\max\{D - x\}].$$

Our Goal

- Find/characterize the optimal manufacturing (ordering), remanufacturing, and disposal strategy so that the total expected (discounted) cost is minimized.

Events Timeline



Formulation

- Type i returns over the periods are $R^i_1, R^i_2, \ldots, R^i_N$, $i = 1, \ldots, K$.
- Let $\mathbf{R}_n = (R^1_n, R^2_n, \ldots, R^K_n)$.
- $(D_n, \mathbf{R}_n)$ can have arbitrary joint distribution, but $(D_1, \mathbf{R}_1), (D_2, \mathbf{R}_2), \ldots, (D_N, \mathbf{R}_N)$ are assumed to be independent.
- There is a discount factor α.

Formulation (Cont'd)

- $I_n =$ inventory level of serviceable product at the beginning of period n;
- J_n^i= inventory level of type i return product at the beginning of period n;
- $\mathbf{J}_n = (J_n^1, \ldots, J_n^K)$;
- $i_n =$ the inventory level of serviceable product after manufacturing and remanufacturing decisions but before demand is realized in period n;

Formulation (Cont'd)

- $j_n^k =$ the inventory level of type k returned product after remanufacturing and disposal decisions but before return occurs in period n;
- $\mathbf{j}_n = (j_n^1, \ldots, j_n^K)$;
- $w_k =$ the remanufacturing quantity of type k return, $k = 1, \ldots, K$;
- $\mathbf{w} = (w_1, \ldots, w_K)$.

Formulation (Cont'd)

- Given $(I_n, \mathbf{J}_n)$, let $V_n(I_n, \boldsymbol{J}_n)$ be the minimum total discounted cost from period n to the end of the planning horizon.

$$
\begin{aligned}
& V_n(I_n, \mathbf{J}_n) \\
= \; & \min_{\boldsymbol{w}, \boldsymbol{j}_n, i_n} \left\{ \sum_{k=1}^{K} r_k w_k + p \left(i_n - I_n - \sum_{k=1}^{K} w_k \right) \right. \\
+ \; & \left. \sum_{k=1}^{K} s_k (j_n^k + \mathsf{E} R_n^k) + G(i_n) + \alpha \mathsf{E} V_{n+1}(i_n - D, \boldsymbol{j}_n + \boldsymbol{R}) \right\}
\end{aligned}
$$

Constraints

- This optimization is subject to constraints
  - $j_n^k \geq 0$, $k = 1, \ldots, K$
  - $0 \leq w_k \leq J_n^k - j_n^k$, $k = 1, \ldots, K$,
  - $\sum_{k=1}^K w_k \leq i_n - I_n$.
- As Simpson (1978), let $V_{N+1}(i, \boldsymbol{j}) = 0$ for any $i, \boldsymbol{j}$.

Single type of returns

- Simpson (1978).
- Simpson's result: Strategy for period n is determined by two numbers: $\xi^0 \geq \xi^1$, such that
  - if initial serviceable inventory level is at least ξ^0, do not manufacture/remanufacture;
  - if initial serviceable inventory level is less than ξ^0, then try to repair to level ξ^0;
  - if after repairing the serviceable product inventory level is less than ξ^1, then manufacture up to ξ^1.

What Happens if Multiple Types of Returns?

- One might want to expect that Simpson's result extends to multiple-type of returns.
- This is not true.
- Under some conditions the control parameters of the optimal strategy is state-independent, but in general, they are not.

System without Disposal

- $w_k = J^k - j^k$.
- Change of variable and let $\mathbf{x} = (x_0, \ldots, x_K)$:

$$
\begin{aligned}
x^0 &= I, \\
x^k &= I + \sum_{\ell=1}^{k} J^\ell, \quad k = 1, \ldots, K, \\
y^0 &= i, \\
y^k &= i + \sum_{\ell=1}^{k} j^\ell, \quad k = 1, \ldots, K
\end{aligned}
$$

Modified Formulation

- Given $\mathbf{x}$, let $V_n(\mathbf{x})$ be the value function.

$$
\begin{aligned}
V_n(\mathbf{x}) &= \min_{\boldsymbol{y}}\{H_n(\boldsymbol{y})\} - r_1 x^0 + \sum_{k=1}^{K-1}(r_k - r_{k+1})x^k \\
&\quad + (r_K - p)x^K
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & x^0 \le y^0 \le y^1 \le \cdots \le y^K, \\
& x^K \le y^K, \\
& y^{k+1} - y^k \le x^{k+1} - x^k, \quad k = 0, \ldots, K-1.
\end{aligned}
$$

$H_n(\boldsymbol{y})$

- H_n is given by

$$
\begin{aligned}
& H_n(\boldsymbol{y}) \\
= \;& (r_1 - s_1)y^0 + G(y^0) \\
& + \sum_{k=1}^{K-1} (r_{k+1} - r_k + s_k - s_{k+1}) y^k \\
& + (p - r_K + s_K) y^K \\
& + \alpha \mathsf{E}[V_{n+1}(y^0 - D, y^1 + R^1 - D, y^2 + R^1 + R^2 - D, \\
& \qquad \ldots, y^K + \boldsymbol{Re}^T - D)].
\end{aligned}
$$

A Technical Result

- **Lemma:** If system parameters satisfy

$$r_1 - s_1 \leq r_2 - s_2 \leq \cdots \leq r_K - s_K, \qquad (1$$

then $V_n(\mathbf{x})$ can be decomposed as

$$V_n(\mathbf{x}) = \sum_{k=0}^{K} Q_n^k(x^k),$$

in which $Q_n^k(\cdot)$ is a univariate convex function for each k.

Theorem

- Under condition (1), the optimal manufacturing/remanufacturing strategy is determined by $K+1$ parameters $\xi^0 > \xi^1 > \cdots > \xi^K$, such that,
- when $\xi^\ell \le x^0 < \xi^{\ell-1}$, then do not use returned product of type $\ell+1, \ldots, K+1$
- $\xi^{K+1} = -\infty, \xi^{-1} = \infty$ and $K+1$ is new product (manufacturing or ordering).

Theorem (Cont'd)

- Repair type 1 to bring inventory level to ξ^0, otherwise, repair type 2 to ξ^1, ..., and the process continues, until, repair (or manufacture) type $\ell + 1$ to ξ^ℓ.
- Illustrate the case $\ell = 0, K + 1$.
- Example $K = 2$.

Illustration I

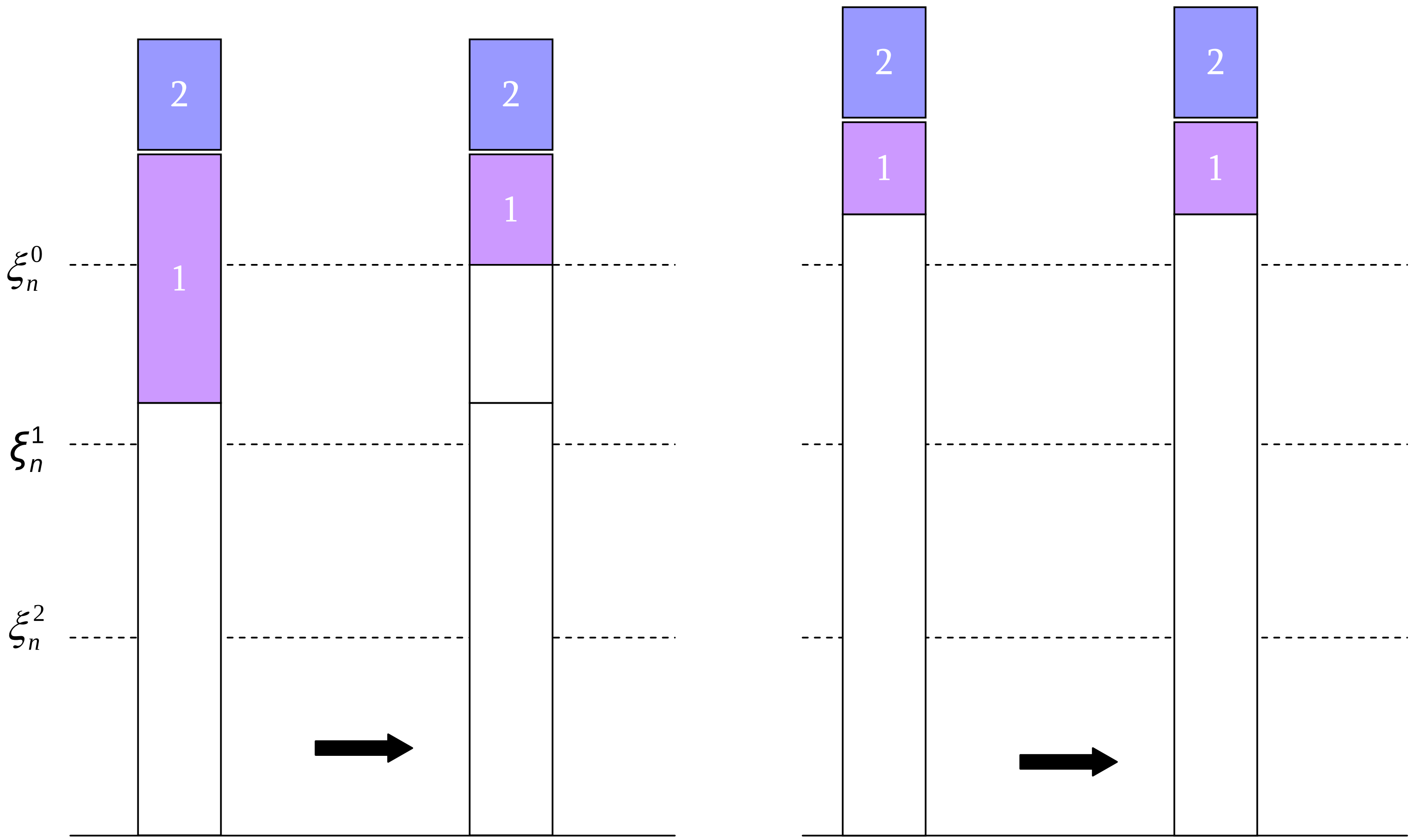


Illustration II

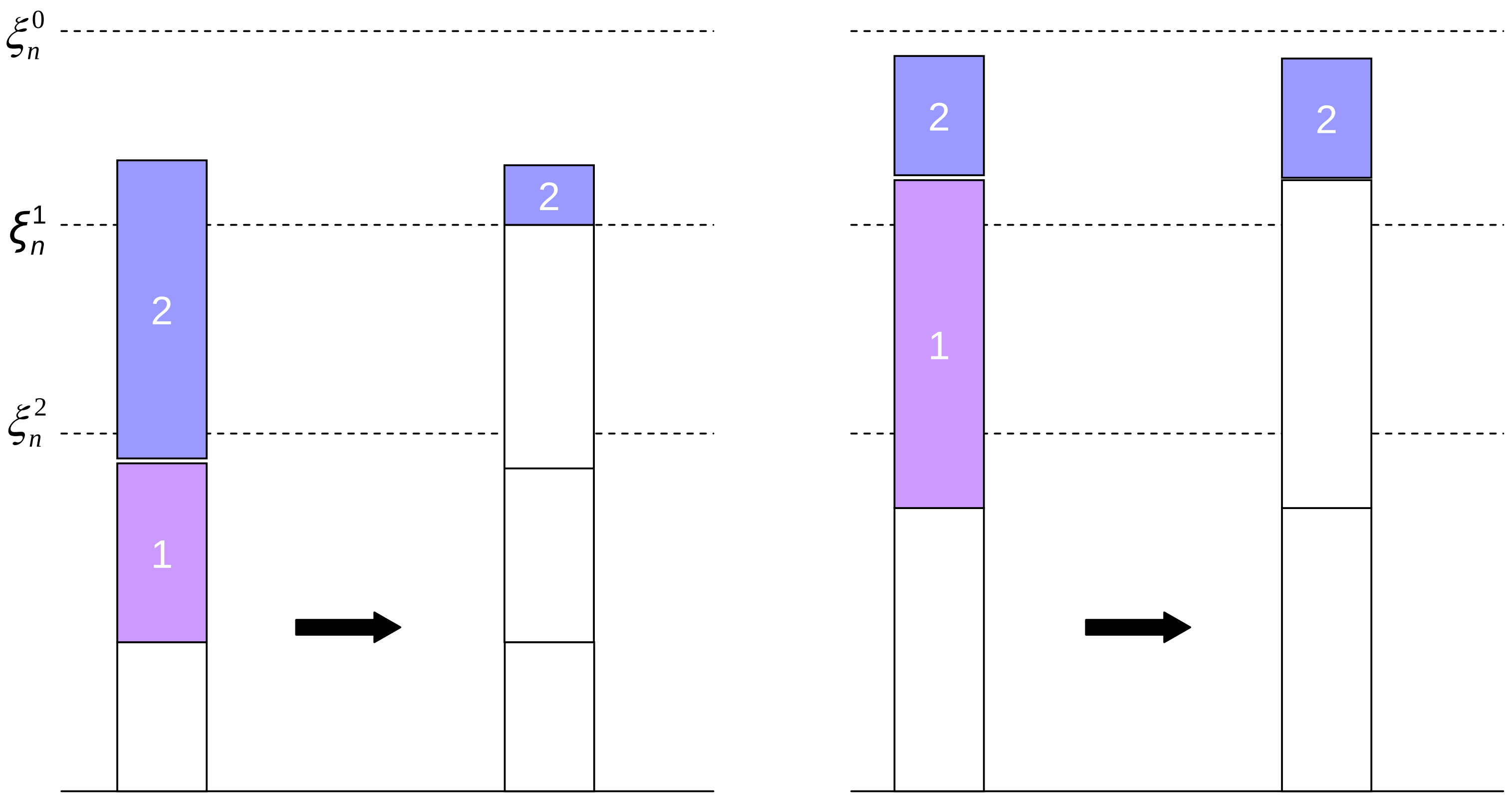
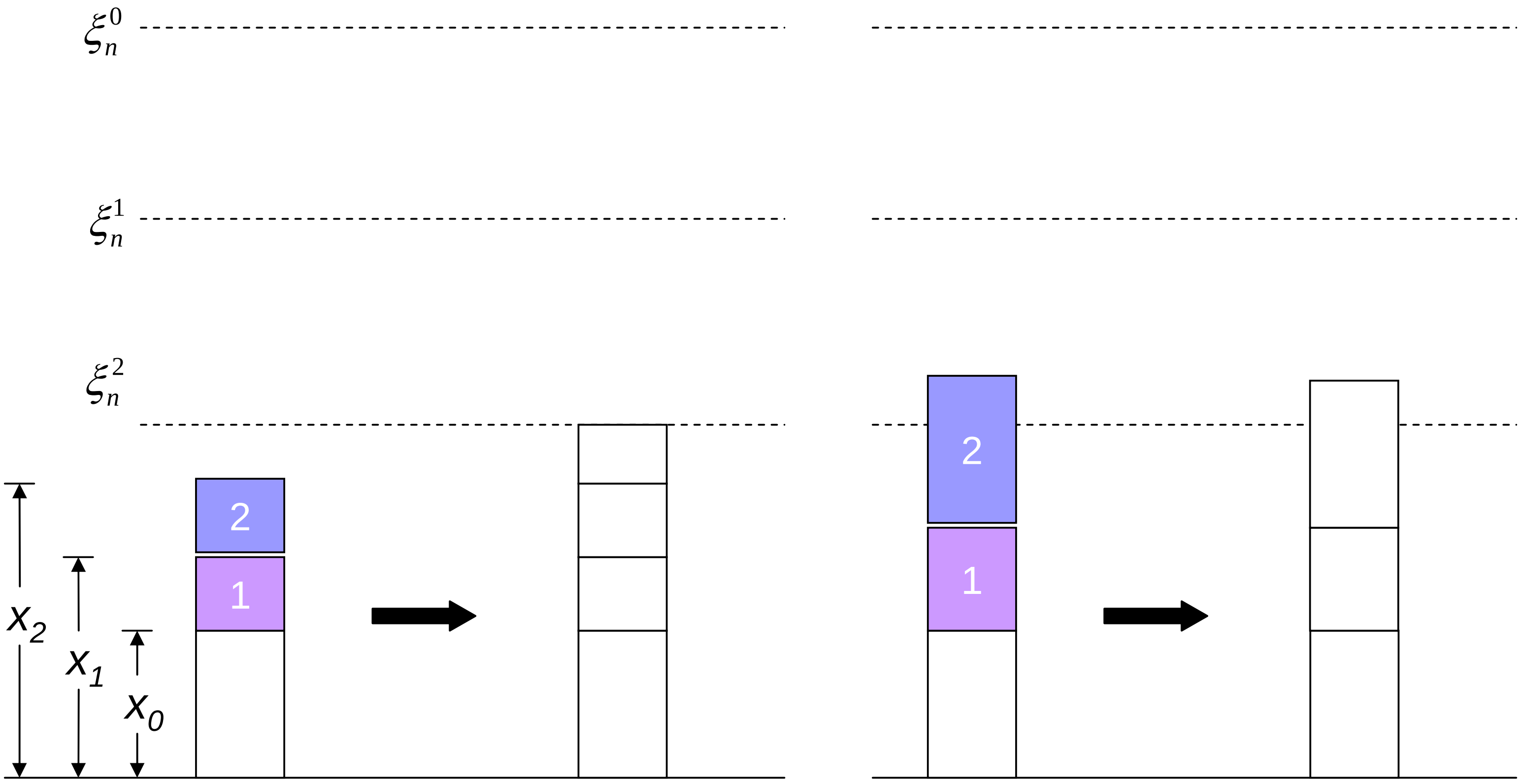


Illustration III



What happens if ...

- What happens if $r_1 - s_1 \leq r_2 - s_2$ is not satisfied?
- The optimal policy will no longer be determined by simple thresholds.
- Example

Example

- $K = 2$, $r_1 = 4, r_2 = 2, s_1 = 2, s_2 = 1, p = 5, \alpha = 1$, $h = 3, b = 5$, $N = 2$. Poisson demand rates 3, and 4.

(x^0, x^1, x^2)	(y^{0*}, y^{1*}, y^{2*})
(4,14,17)	(12,14,17)
(4,15,16)	(13,15,16)
(4,15,17)	(13,15,17)
(4,15,18)	(12,15,18)
(4,15,19)	(12,15,19)

What is optimal, then?

- Suppose $r_1 - s_1 > r_2 - s_2$.
- $H_n(\mathbf{x})$ is no longer decomposable.
- We can characterize the optimal policy, which is complicated with state-dependent control parameters.
- We also develop simple heuristic policies.

Systems with Disposals

- Suppose there exists an M, for $k \geq M$, type k returns can be disposed.
- Under stronger condition $s_1 \leq \cdots \leq s_K$, the optimal policy is determined by a set of control parameters.
- Otherwise the optimal policy can be characterized, and it is complicated with state-dependent control parameters.

Theorem

- Under condition (2), the optimal remanufacturing/manufacturing and disposal policy for period n, is determined by two sets of parameters $\{\xi^k, k = 0, \ldots, K\}$ and $\{\eta^k, k = M, \ldots, K\}$, satisfying

$$\xi^K \;\leq\; \cdots \leq \xi^1 \leq \xi^0, \quad \text{and} \quad \eta^K \leq \cdots \leq \eta^M,$$

and

$$\xi^k \;\leq\; \eta^{k+1}, \quad k = M-1, \ldots, K-1.$$

Illustration IV

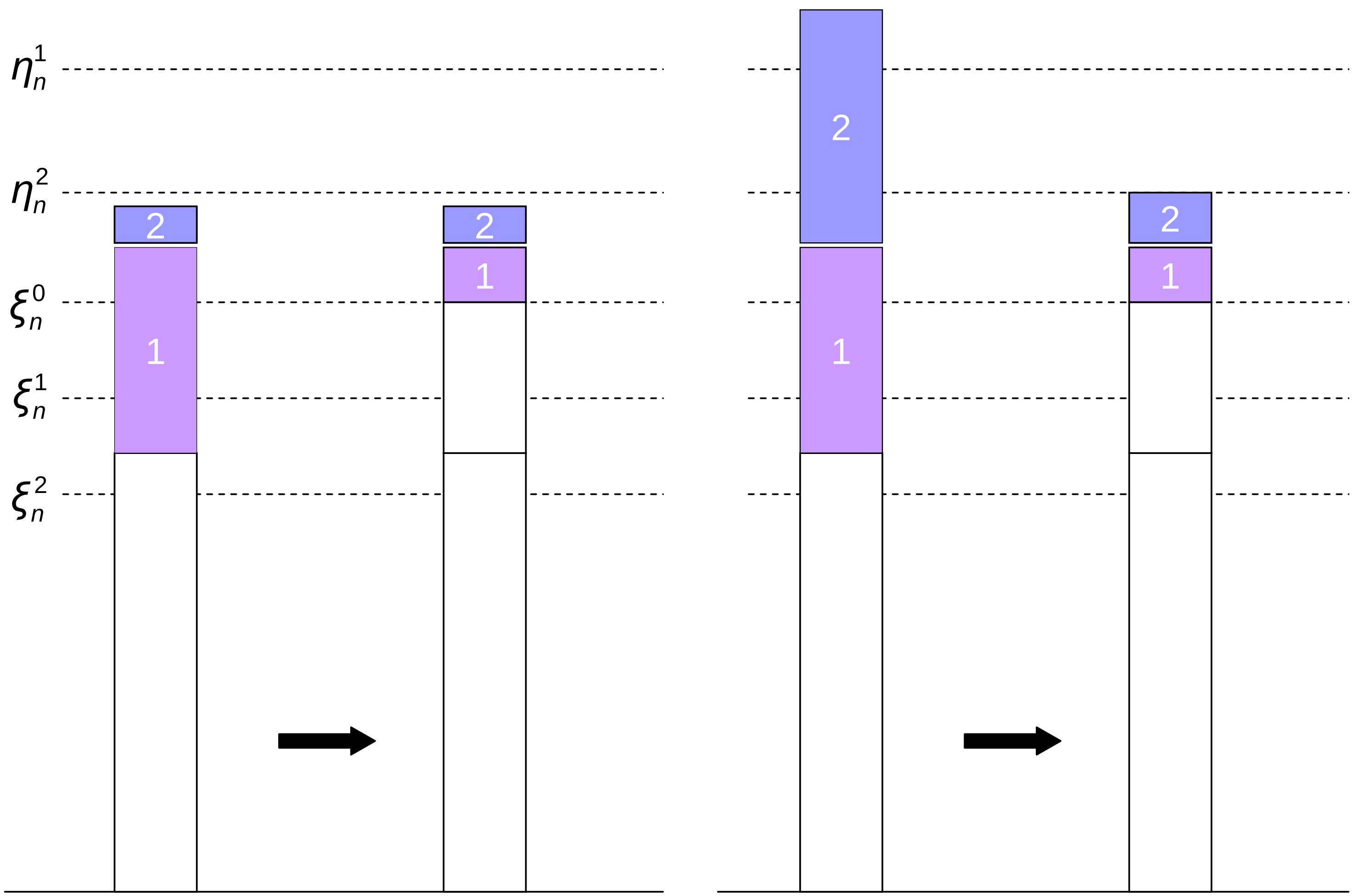
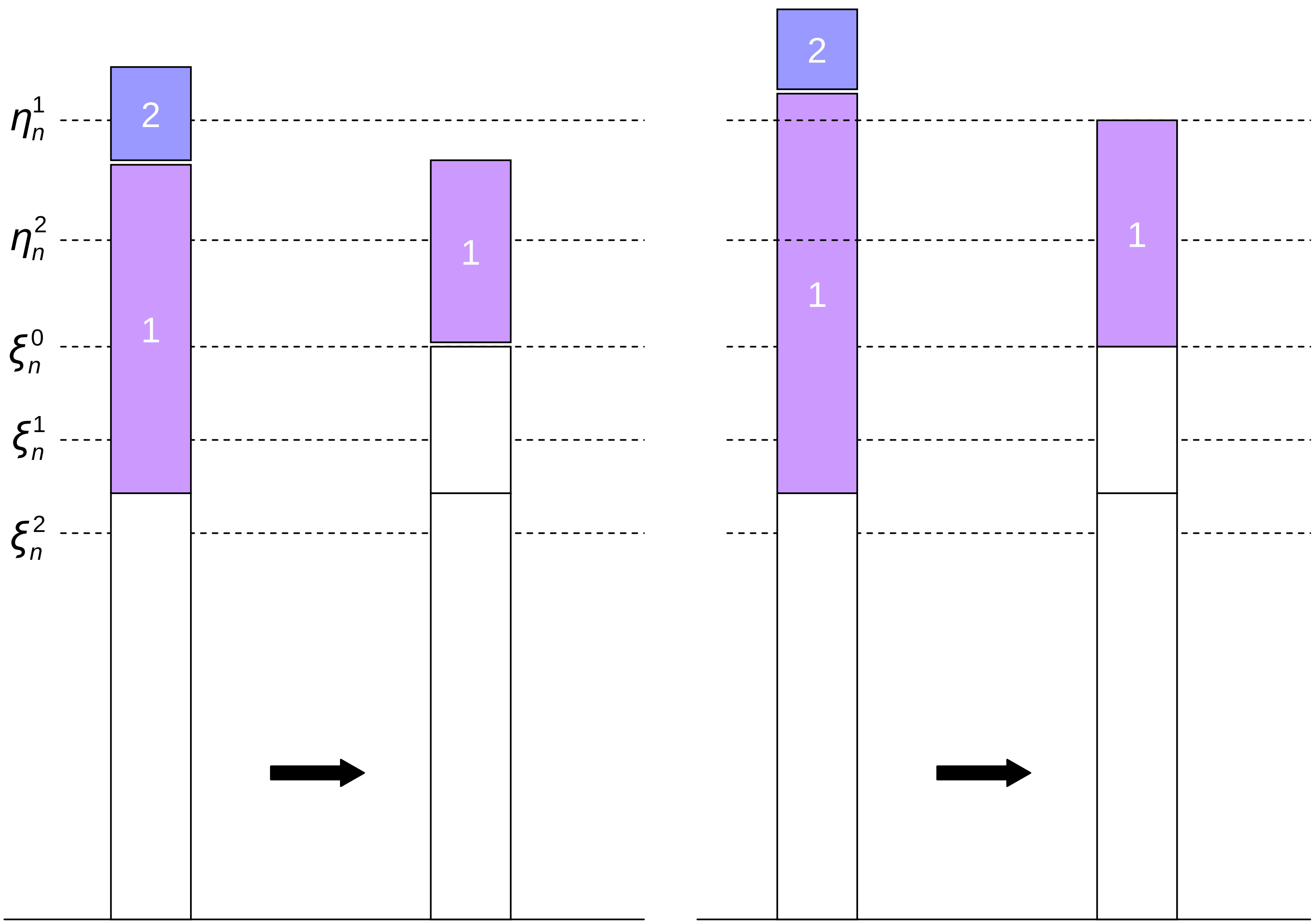


Illustration V



Then What?

- Thus, only under conditions (1) and (2) the optimal policy has a simple form.
- If these conditions are not satisfied, optimal policy is complicated and state-dependent.
- We develop simple heuristic policies with state-independent control parameters.

Heuristic I

- Illustrate the heuristic solution for $K = 2$.
- Suppose the data is stationary.

$$\xi^0 = \overline{F}_D^{-1}\left(\frac{(1-\alpha)r_1 - s_1 + h}{h+b}\right),$$

$$\xi^1 = \overline{F}_D^{-1}\left(\frac{(1-\alpha)r_2 - s_2 + h}{h+b}\right),$$

$$\xi^2 = \overline{F}_D^{-1}\left(\frac{(1-\alpha)p + h}{h+b}\right).$$

Heuristic I (Cont'd)

$$
\begin{aligned}
& s_1 + \alpha(r_1 - r_2)\Bigg[P(\eta^1 - D + R^1 \le \xi^1) \\
& + \mathsf{E}\left[\frac{D - R^1}{\eta^1 - \xi^1}\mathbf{1}(\xi^1 < \eta^1 - D + R^1 < \eta^1)\right]\Bigg] \\
& + \alpha(r_2 - p)\Bigg[P(\eta^2 - D + R^1 + R^2 \le \xi^2) \\
& + \mathsf{E}\left[\frac{\eta^2 - \eta^1 + D - R^1 + R^2}{\eta^1 - \xi^2}\mathbf{1}(\xi^2 < \eta^1 - D + R^1 + R^2 < \eta^2)\right. \\
& = 0
\end{aligned}
$$

Heuristic I (Cont'd)

$$
\begin{aligned}
& s_2 + \alpha(r_2 - p)P(\eta^2 - D + R^1 + R^2 \leq \xi^2) \\
& +\alpha(r_2 - p)\mathsf{E}\left[\frac{D - R^1 + R^2}{\eta^2 - \xi^2}\mathbf{1}(\xi^2 < \right. \\
& \left. \eta^2 - D + R^1 + R^2 < \eta^2)\right] \\
= \quad & 0.
\end{aligned}
$$

Heuristic II

- ξ^1 and ξ^2 are determined jointly with η^1 and η^2 by solving

$$
\begin{aligned}
&(r_2 - s_2) + G'(\xi^1) - \alpha r_1 + \alpha(r_1 - r_2)\Bigg[P(R^1 - D \leq 0) \\
&+\mathsf{E}\left[\frac{\eta^1 - \xi^1 + D - R^1}{\eta^1 - \xi^1}\mathbf{1}(\xi^1 < \xi^1 - D + R^1 < \eta^1)\right]\Bigg] \\
&= 0.
\end{aligned}
$$

Heuristic II (Cont'd)

$$p + G'(\xi^2) - \alpha r_1 + \alpha(r_1 - r_2)\Bigg[P(\xi^2 - D + R^1 \le \xi^1) +$$

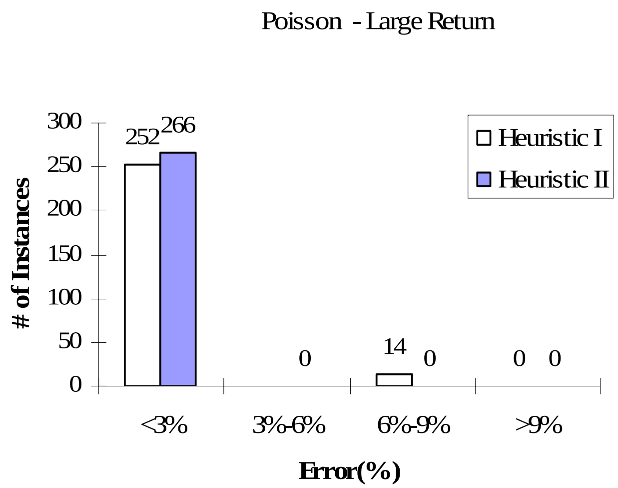
$$\mathsf{E}\left[\frac{\eta^1 - \xi^2 + D - R^1}{\eta^1 - \xi^1}\mathbf{1}(\xi^1 < \xi^2 - D + R^1 < \eta^1)\right]\Bigg]$$

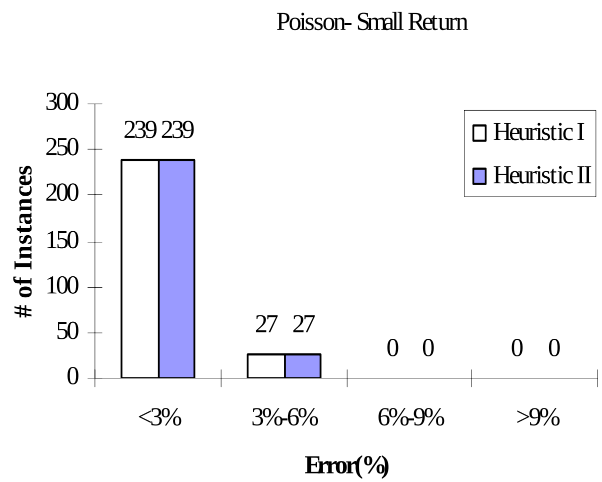
$$+\alpha(r_2 - p)\Bigg[P(-D + R^1 + R^2 \le 0)$$

$$+\mathsf{E}\left[\frac{\eta^2 - \xi^2 + D - R^1 + R^2}{\eta^2 - \xi^2}\mathbf{1}(\xi^2 < \xi^2 - D + R^1 + R^2 < \eta^2)\right]\Bigg]$$

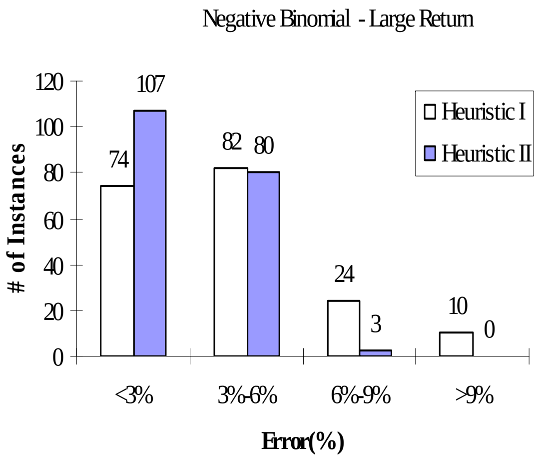
$$= 0.$$

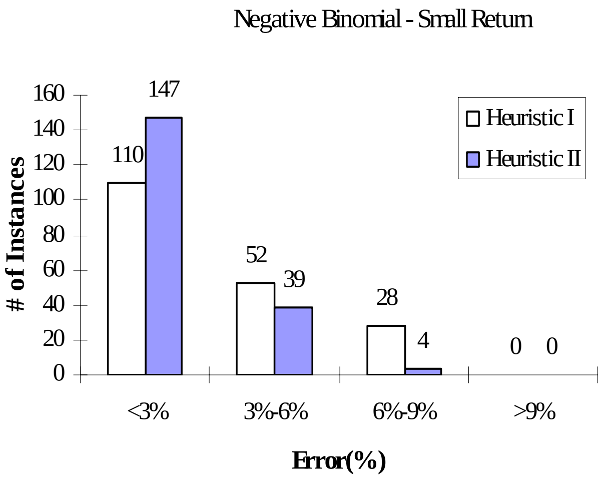
Numerical Studies I





Numerical Studies II





Performance of Heuristic

	average error(%)	maximum error
Poisson sm return	1.22	4.78
Neg-Binomial sm return	1.36	6.86
Poisson lg return	0.98	1.77
Neg-Binomial lg return	2.67	8.28

Conclusion

- Inventory systems with multiple types of returned products, and with or without disposals.
- Characterize the optimal remanufacturing/manufacturing and disposal policies
- In some scenarios, simple and state-independent policy is optimal
- In others, complicated and state-dependent
- Heuristics are developed and tested numerically.

Thank You …

For Your Attention!