1. (i) [10%] Find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) for \( \ln(x + yz) = 1 + xy^2z^3 \).
(ii) [10%] Find the curvature of the curve \( r(t) = \sin ti + \cos tj + \sin tk \).

2. (i) [10%] Compute the gradient of the function \( \frac{r}{\sin r} \) where \( r \) is the distance from \( (x, y, z) \) to the origin.
(ii) [10%] Show that the function \( f(x, y) = \frac{x^2 - y^2}{x^2 + y^2} \) does not have limit at \( (0, 0) \).

3. (i) [6%] Find the equation of the tangent plane for \( z = \sin x + \sin y + \sin (x + y) \) at \( (0, 0, 0) \).
(ii) [14%] Find the absolute maximum and minimum values of \( f(x, y) = e^{-x^2 - y^2}(x^2 + 2y^2) \) on the disc \( x^2 + y^2 \leq 4 \).

4. (i) [10%] Find the mass and center of mass of solid hemisphere of radius \( a \) (i.e., above the \( xy \)-plane and below the sphere of radius \( a \)) if the density at any point is proportional to its distance from the \( xy \)-plane.
(ii) [10%] Find the volume of the solid \( T \) enclosed by the ellipsoid \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \).

5. (i) [10%] A function \( f(x, y) \) is called homogeneous of degree \( n \) if \( f \) has continuous second-order partial derivatives and \( f(tx, ty) = t^n f(x, y) \) for all \( t \). Use the chain rule to show that if \( f \) is homogeneous of degree \( n \), then
\[
x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = nf(x, y).
\]
(ii) [10%] Compute \( \int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} \, dx \, dy \).