1. [10%] Let $F(x, y) = x^3 y^4 \mathbf{i} + x^4 y^3 \mathbf{j}$. Find a function $f$ such that $\nabla f = F$ and compute the line integral $\int_C F \cdot d\mathbf{r}$ where $C$ is the curve $\mathbf{r}(t) = \sqrt{t} \mathbf{i} + (1 + t^3) \mathbf{j}$, $0 \leq t \leq 1$.

2. [10%] Use Stoke’s theorem to compute the integral $\iint_S \text{curl} \mathbf{F} \cdot \mathbf{n} \, dS$ where $\mathbf{F}(x, y, z) = yz \mathbf{i} + xz \mathbf{j} + xy \mathbf{k}$ and $S$ the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the $xy$-plane.

3. [10%] Find the flux of the vector field $\mathbf{F}(x, y, z) = x \mathbf{i} + y \mathbf{j} + y^2 \mathbf{k}$ over the unit sphere $x^2 + y^2 + z^2 = 1$.

4. [10%] Use Green’s theorem to find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for some $a, b > 0$.

5. [10%] Sketch the solid whose volume is given by the iterated integral $\int_0^2 \int_0^{2-x} \int_0^{1-x^2} dz \, dy \, dx$.

6. [10%] Find local maximum and minimum values and saddle points of the function $f(x, y) = x^2 + y^2 + x^2 y + 4$.

7. (i) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ for $xy^2 + yz^2 + zx^2 = 3$.
   (ii) Find a unit normal vector of the plane passing through the point $(0, 0, 1)$ and spanned by the two vectors $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} - 2\mathbf{k}$.

8. [10%] Find the length of the curve $\mathbf{r}(t) = t^2 \mathbf{i} + 2t \mathbf{j} + \ln t \mathbf{k}$ for $1 \leq t \leq e$.

9. [10%] Test the series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$ for absolutely convergence, conditionally convergence or divergence.

10. [10%] Let $f(x) = \frac{e^x - 1}{x}$.
   (1) Find the power series representation of $f$ in powers of $x$.
   (2) Differentiate the power series in (1) and show that $\sum_{n=1}^{\infty} \frac{n}{(n + 1)!} = 1$. 

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**FINAL FOR CALCULUS**

**Time:** 8:10–10:00 AM, Friday, June 22, 2001

**Instructor:** Shu-Yen Pan

No calculator is allowed. No credit will be given for an answer without reasoning.