Time: 8:10–9:55 AM, Friday, December 15, 2000
Instructor: Shu-Yen Pan

No calculator is allowed. No credit will be given for an answer without reasoning.

1. Find
   (1) [4%] \( \int \frac{1}{x^2 + 4x + 3} \, dx \).
   (2) [4%] \( \int \frac{1}{x^2 + x} \, dx \).

2. (1) [4%] Evaluate \( \int_{-1}^{2} |2x + 1| \, dx \).
   (2) [4%] Integrate \( \int \tan^4 x \sec^4 x \, dx \).

3. (1) [4%] Find \( f' \) if \( f(x) = (x^2)^x \).
   (2) [4%] Integrate \( \int \coth x \, dx \).

4. (1) [4%] Find the exact value of the expression \( \sin(\cos^{-1} \frac{3}{5}) \).
   (2) [4%] Find the area of the region bounded by the curves \( y = 20 - x^2 \) and \( y = x^2 - 12 \).

5. [6%] Use Newton method with the specified initial approximation \( x_1 = -1 \) to find \( x_2 \), the second approximation to the root of the equation \( x^3 + x + 1 = 0 \).

6. [6%] Suppose that \( f \) is differentiable, \( f(0) = 0 \), \( f(1) = 1 \), \( f'(x) > 0 \) and \( \int_0^1 f(x) \, dx = \frac{1}{4} \). Find the value of the integral \( \int_0^1 f^{-1}(y) \, dy \).

7. [8%] Let \( f(x) = \int_2^x \sqrt{1 + t^2} \, dt \).
   Prove that \( f(x) \) has an inverse and find \( f'(0) \).

8. [8%] Find the integral \( \int_0^2 \frac{x^2}{(x+4)^2} \, dx \).

9. [8%] Find the limit \( \lim_{n \to \infty} \frac{1}{n} \left( \sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}} \right) \).

10. [8%] A fence 3 meters tall runs parallel to a tall building at a distance of 2 meters from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

11. [8%] If \( x \sin x = \int_0^x f(t) \, dt \), where \( f \) is a continuous function, find \( f(4) \).

12. [8%] Find the limit \( \lim_{x \to \infty} \left( 1 + \frac{3}{x} \right)^{2x} \).

13. [8%] Let \( f \) be a function such that \( f' \) is continuous on \([a, b]\). Prove that \( \int_a^b f(t)f'(t) \, dt = \frac{1}{2} \left( f(b) - f(a) \right) \left( f(b) + f(a) \right) \).