1. (i) [5%] Suppose that (0, 2) is a critical point of a function \( g \) with continuous second derivatives. Suppose that \( g_{xx}(0, 2) = -1, g_{xy}(0, 2) = 2 \) and \( g_{yy}(0, 2) = -8 \). Use second derivative test to classify the critical point (0, 2).

(ii) [5%] Find an equation of the tangent plane to the surface \( z = e^x \ln y \) at the point (3, 1, 0).

2. [10%] Let \( u = x + at \) and \( v = x - at \). Then use chain rule to show that any differentiable function of the form

\[
z = f(x + at) + g(x - at)
\]

is a solution of the wave equation

\[
\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}.
\]

3. [10%] Find the directional derivative of the function \( g(x, y, z) = z^3 - x^2y \) at the point (1, 6, 2) in the direction \( \mathbf{v} = 3i + 4j + 12k \).

4. [20%] Find the extreme values of the function \( f(x, y) = e^{-xy} \) on the region \( x^2 + 4y^2 \leq 1 \).

5. [10%] Evaluate

\[
\int_0^{\pi/2} \int_0^{\pi/2} \sin(x + y) \, dy \, dx.
\]

6. [10%] Find the area of the part of the paraboloid \( z = x^2 + y^2 \) that lies under the plane \( z = 4 \).

7. [10%] Use triple integral to show that the volume of the solid bounded by a sphere of radius \( a \) is \( \frac{4}{3}a^3\pi \).

8. [10%] The average value of a function \( f(x, y, z) \) over a solid region \( E \) is defined to be

\[
f_{ave} = \frac{1}{V(E)} \iiint_E f(x, y, z) \, dV
\]

where \( V(E) \) is the volume of \( E \). Find the average value of the function \( f(x, y, z) = x + y + z \) over the tetrahedron with vertices (0, 0, 0), (1, 0, 0), (0, 1, 0) and (0, 0, 1).

9. [10%] Evaluate the integral

\[
\int_0^1 \int_0^1 \sqrt{x^3 + 1} \, dx \, dy
\]

by reversing the order of integration.