MIDTERM 1 FOR CALCULUS

Time: 09:10–11:00, Tuesday, Nov 19, 2002
Instructor: Shu-Yen Pan

No credit will be given for an answer without reasoning.

1. (1) [5%] Find an equation of the tangent line to the curve \( y = \frac{x^2}{x-2} \) at the point (1, 1).
   (2) [5%] Find the limit \( \lim_{s \to 16} \frac{4-\sqrt{s}}{s-16} \).

2. (1) [5%] Suppose that \( f \) and \( g \) are functions that are differentiable at \( x = 1 \) and that \( f(1) = 2 \), \( f'(1) = -1 \), \( g(1) = -2 \) and \( g'(1) = 3 \). Find the value \( h'(1) \) where \( h(x) = \frac{xf(x)}{x+g(x)} \).
   (2) [5%] Find \( \frac{dp}{dt} \) if \( p = \sqrt{x} + \sqrt{x} \).

3. (1) [5%] Given the graph of \( y = f(x) \) below, sketch the graph of \( y = f'(x) \).
   (2) [5%] A city’s population (in thousands) \( t \) years from now is estimated to be \( P(t) = \frac{25t^2 + 125t + 100}{t^2 + 5t + 40} \). At what rate will the population be increasing 10 years from now?

4. (1) [5%] Use second derivative test to find the relative extrema of the function \( g(x) = x^2 + \frac{1}{x} \).
   (2) [5%] Use the definition of derivative to prove that the function \( f(x) = x^{3/5} \) is not differentiable at \( x = 0 \)

5. (1) [5%] Let \( f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ ax + b & \text{if } x > 1 \end{cases} \)
   Find the values of \( a \) and \( b \) so that \( f \) is continuous and has a derivative at \( x = 1 \).
   (2) [5%] When living things die, the carbon 14 in their body decays exponentially into ordinary carbon. The proportion of carbon 14 remaining after \( t \) years is \( e^{-kt} \). We know that the half-life of carbon 14 is 5770 years. Find the constant \( k \).

6. [10%] Find the first and second derivatives (i.e., \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \)) of the function defined implicitly by the equation \( \sqrt{xy} = x + y \).

7. [10%] Find the dimensions of the top-open cylindrical tin can with volume 60\( \pi \) cubic centimeters that can be made from the least amount of tin.

8. [10%] Which is better? 10.2% interest compounded annually, 10% interest compounded quarterly, or 9.8% interest compounded continuously?

9. [20%] Sketch the graph of \( y = \sqrt{x^2 + 1} \). (You have to provide the following information: domain, \( x \) and \( y \) intercepts, vertical and horizontal asymptotes, the intervals where the function is increasing, decreasing, concave up, concave down, local extreme values, inflection points.)