No credit will be given for an answer without reasoning.

1. (i) [5%] Find $f_y(8, -6)$ for $f(x, y) = \sqrt{x^2 + y^2}$.
(ii) [5%] Find the limit $\lim_{x \to \infty} \frac{x}{\ln x}$.

2. (i) [5%] Find the constant $a$ such that the function $f(x) = ax^2(1 - x)$ is a probability density function on the interval $[0, 1]$.
(ii) [5%] Keep the assumption in (i). Find the probability $P(0 \leq x \leq 1/2)$.

3. (i) [5%] Evaluate the double integral by changing the order of integration:
$$\int_0^1 \int_0^1 \sin \left( \frac{y^3 + 1}{2} \right) dy \, dx.$$ 
(ii) [5%] Let $f$ be a differentiable function such that $f(0) = 4$, $f(3) = 5$, and $f'(3) = 5$. What is $\int_0^3 xf''(x) \, dx$?

4. (i) [5%] Evaluate the integral $\int \frac{\sin t \cos t}{\sqrt{1 + \cos^2 t}} \, dt$.
(ii) [5%] Find the improper integral $\int_{-\infty}^{\infty} \frac{e^x}{(1 + e^x)^2} \, dx$.

5. Consider the differential equation: $y' = y(1 - y)$.
(i) [5%] Find the general solution $y(t)$.
(ii) [5%] Find the solution for the equation with initial condition $y(0) = 1/2$.

6. (i) [5%] A ball is dropped from a height of 16 meters. Each time it drops $h$ meters, it rebounds $0.6h$ meters. Find the total vertical distance travelled by the ball.
(ii) [5%] Determine the convergence or divergence of the series
$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3}}.$$
7. Estimate \( \sqrt[3]{67} \) by the following two methods:
   (i) [5\%] Apply Newton’s method for two iterations to the function \( f(x) = x^3 - 67 \) with initial guess \( x_1 = 4 \).
   (ii) [5\%] Use the degree-two Taylor polynomial centered at \( x = 64 \) to approximate the function \( g(x) = x^{1/3} \). Then evaluate the Taylor polynomial at \( x = 67 \).

8. (i) [5\%] Consider a random variable \( X \) with probability
   \[
P(X = k) = e^{-3} \frac{3^k}{k!}
   \]
   for \( k = 0, 1, 2, 3, \ldots \). Find the expected value \( E[X] \).
   (ii) [5\%] Find the standard deviation \( \sigma \) of the probability density function \( f(x) = 4e^{-4x} \), \( 0 \leq x < \infty \).

9. [10\%] Find the Taylor series centered at \( x = 0 \) of the function
   \[
f(x) = \ln \left( \frac{1 + x}{1 - x} \right).
   \]
   Then find the radius of convergence of the Taylor series. [Hint: \( f(x) = \ln(1 + x) - \ln(1 - x) \)]

10. [10\%] Assume that the temperature \( T \) at a point \((x, y, z)\) on the sphere \( x^2 + y^2 + z^2 = 1 \) is given by
    \[
    T(x, y, z) = 10xy^2z.
    \]
    Use Lagrange multiplier method to find the point(s) on the sphere at which the temperature is greatest and the point(s) at which it is least.