Ph'D Qualifying Exam
General Analysis

Oct. , 2016

E:Easy, M:Moderate, D:Difficult

1. (15 pts, E) $\{E_k\}$ is an increasing sequence of sets in R^n and $E_k \to E$. Please show that $\limsup_{k\to\infty} E_k = \liminf_{k\to\infty} E_k = E$.

2. (15 pts, E) Let f be defined and measurable in R^n. If T is a nonsingular linear transformation of R^n, show that $f(Tx)$ is measurable.

3. (15 pts, M) Use Fubini's theorem to prove that

$$\int_{R^n} e^{-|x|^2} dx = \pi^{n/2}$$

4. (15 pts, E, 2014) Let E be a measurable set in R^n. f and f_k are measurable in E. $\int_E |f - f_k|^2 \to 0$ as $k \to \infty$. Please prove that f_k converges to f in measure.

5. (20 pts, M, 2012) Please prove or disprove
   (a) If f is strictly increasing continuous function with $f'(x) = 0$ a.e., then f is a constant function.
   (b) If f is an absolutely continuous function with $f'(x) = 0$ a.e., then f is a constant function.

6. (20 pts, M, 2015) Let $1 \le p < \infty$ and g be an integrable function defined on $[0,1]$. Suppose that there exists $M > 0$ such that

$$\left|\int_0^1 fg dx\right| \le M\|f\|_p$$

for all bounded measurable functions f. Please prove that $\|g\|_q \le M$, where $1/p + 1/q = 1$.