Name and Student ID: _____

Homework 10, Analytic Geometry and Matrices

Properties of Invertible Transformations/Matrices

1. Can a $m \times n$ matrix, with $m \neq n$, be invertible? Answer with sufficient supporting reasons.

2. Prove that if $T^2 = T \circ T = 0$, then T is not invertible.

Computations of Inverses

1. Find the inverse matrices of
   (a) $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$.
   (b) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.
   (c) $\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 2 \\ 2 & 0 & 1 & 0 \\ 0 & -1 & 1 & -3 \end{pmatrix}$.

2. Compute the ranks of the following matrices
   (a) $\begin{pmatrix} 0 & -2 & 4 \\ 1 & 1 & -1 \\ 2 & 4 & -5 \end{pmatrix}$.
   (b) $\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ -2 & -3 & 0 & 3 \\ 3 & 4 & -2 & -3 \end{pmatrix}$.

3. Express the invertible matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

   as a product of elementary matrices.