Name and Student ID: _____

Homework 11, Analytic Geometry and Matrices

Rank Computations/Row Reduced Echelon Forms

1. Find the rank of the following matrices:

   (a) $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \end{pmatrix}$.

   (b) $\begin{pmatrix} 1 & 2 & 3 & 1 & 1 \\ 1 & 4 & 0 & 1 & 2 \\ 0 & 2 & -3 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$.

2. Find all values of a so that

$$\begin{pmatrix} 1 & 2 & a \\ 1 & 1 & 2a \\ 0 & 1 & 3 \end{pmatrix}$$

   is of

   (a) rank 2.

   (b) rank 3.

   Can this matrix be of rank 1?

3. Let $A \in Mat_{m \times n}$, and c a nonzero real number. Prove that rank(cA)=rank(A).

4. Let $A \in Mat_{m \times n}$ with rank m. Prove that there is $B \in Mat_{n \times m}$ so that $AB = I_m$, the $m \times m$ identity matrix.

5. Find the row echelon form of

$$A = \begin{pmatrix} 2 & 3 & 1 & 4 & -9 \\ 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & 2 & -5 \\ 2 & 2 & 2 & 3 & -8 \end{pmatrix}.$$

   Also, compute the nullity of A.

Systems of Linear Equations

1. Solve the following systems of linear equations using Gaussian eliminations.

(a)
$$\begin{cases} x_1 - 2x_2 - x_3 = 1 \\ 2x_1 - 3x_2 + x_3 = 6 \\ 3x_1 - 5x_2 = 7 \\ x_1 + 5x_3 = 9 \end{cases}$$

(b)
$$\begin{cases} 2x_1 - 2x_2 - x_3 + 6x_4 - 2x_5 = 1 \\ x_1 - x_2 + x_3 + 2x_4 - x_5 = 2 \\ 4x_1 - 4x_2 + 5x_3 + 7x_4 - x_5 = 6 \end{cases}$$