Name and Student ID: _____

Homework 1 Supplementary Problems

1. What is wrong with the "set"

$$A = \{x \mid x \notin x\}?$$

Is anything of the form $\{x \mid P(x)\}$ a set? (Please google "Russell's paradox").

2. Consider the unit sphere

$$\mathbb{S}^2 := \{(x, y, z) \mid x^2 + y^2 + z^2 = 1\} \subset \mathbb{R}^3.$$

and the "north pole" $N = (0, 0, 1) \in \mathbb{S}^2$. Consider the mapping

$$\Phi : \mathbb{S}^2 \backslash N \to \mathbb{R}^2$$

defined by

$$\Phi(x, y, z) = \frac{(x, y)}{1 - z}.$$

   (a) Is Φ well defined on it domain?

   (b) Place the center of $\mathbb{S}^2$ at the origin $(0, 0, 0)$. For every $(x, y, z) \in \mathbb{S}^2 \backslash N$, write down the parametric equation of the line l going through N and (x, y, z).

   (c) What is the point of intersection between l and xy plane (ie. $z = 0$)? Any relation to $\Phi(x, y, z)$?

   (d) Explain, loosely using languages introduced in class and perhaps some drawing, that Φ defines a coordinate for $\mathbb{R}^2$.

This is the well known "*stereographic projective coordinates*" of $\mathbb{R}^2$, globally defined for all $\mathbb{R}^2$. For 10 point extra credit, construct the stereographic projective coordinates for $\mathbb{R}^n$.