Name and Student ID: ____

Homework 2, Analytic Geometry and Matrices

Problems from (or similar to) *Thomas' Calculus*:

1. Sketch the traces given by the following equations:
   (a) $x = 2$ and $y = 3$
   (b) $z = x$
   (c) $x^2 + y^2 = 4$ and $z = y$
   (d) $x^2 + y^2 + z^2 = 4$ and $y = x$

2. Sketch the traces given by the following combinations of equations and inequalities:
   (a) $x^2 + z^2 \leq 4$
   (b) $z \leq y$
   (c) $x^2 + y^2 + z^2 \leq 1$ and $z \geq 0$
   (d) $y \geq x^2$ and $z \geq 0$

3. Find an equation for the set of all points equidistant from the point $(0, 0, 2)$ and the $xy-$plane.

Problems from class:

1. For a limoçon, that is, image of the curve $r = a \pm b \sin\theta$ or $r = a \pm b \cos\theta$ under the mapping Φ for polar coordinate, there are four possible geometric shapes: (A)limaçon with an inner loop, (B)heart, (C)dimpled limaçon, and (D)convex limaçon. The corresponding shape is determined by the value $\left|\frac{a}{b}\right|$. Prove that, (A) appears when $\left|\frac{a}{b}\right| \in (0, 1)$, (B) appears when $\left|\frac{a}{b}\right| = 1$, (C) appears when $\left|\frac{a}{b}\right| \in (1, 2)$, and (D) appears when $\left|\frac{a}{b}\right| \geq 2$. For (C) and (D) just prove the case $y = a + b\cos\theta$ with $a > b > 0$. (Hint for (C) and (D): Observe the fundamental difference between dimpled and convex curves. Pick arbitrary points on the curves and draw vertical lines through them. These lines have quite different relationships with the curves (C) and (D). Try to describe the differences resulting from different values $\left|\frac{a}{b}\right|$.)

2. For a limoçon, that is, image of the curve $r = a \pm b\sin\theta$ or $r = a \pm b\cos\theta$ under the mapping Φ for polar coordinate, discuss the orientation of the images according to four different situations: $b\sin\theta$, $b\cos\theta$, where $b > 0$, or $b < 0$.

3. For rose curves $r = a\sin(n\theta)$ or $r = a\cos(n\theta)$, prove that there are n petals if n is odd and $2n$ petals if n is even.