Name and Student ID: _____

Homework 3, Analytic Geometry and Matrices

Problems concerning coordinates:

1. Consider cylindrical coordinate $\Phi : r\theta u \to xyz$. Sketch the pre-image, in $r\theta u$ space, of the region bounded by $x^2 + y^2 \leq 1$, $y \geq 0$, $y \leq x$, and $0 \leq z \leq 1$ in xyz space. (without concern on the boundary requirement for one-to-one property of Φ):

2. Consider spherical coordinate $\Phi : \rho\theta\phi \to xyz$. Sketch the pre-image, in $\rho\theta\phi$ space, of the lower half of the ball $x^2 + y^2 + z^2 \leq 1$ in xyz space. (without concern on the boundary requirement for one-to-one property of Φ):

Problems concerning conic sections:

1. Given $\phi \in \mathbb{R}$, prove, using polar coordinates, that the map

$$R(x, y) = (x\cos\phi - y\sin\phi\ ,\ x\sin\phi + y\cos\phi)$$

   rotates every point on $\mathbb{R}^2$ by an angle ϕ, counterclockwise. What is the inverse of R?

2. Write down a rotation $R : \mathbb{R}^2 \to \mathbb{R}^2$ that takes two opposite points $\pm(\alpha, \beta)$ to two opposite points on $\pm(c, 0)$ on x-axis. What is c?

3. Using previous two problems, write down the equation of the ellipse with foci $\pm(\alpha, \beta)$ and length of major axis $2a$ for some $a \geq \sqrt{\alpha^2 + \beta^2}$.

4. Write down the equation of an ellipse with general foci (p, q), and (r, s) and length of major axis $2a$ for some $a \geq \frac{1}{2}\sqrt{(p-r)^2 + (q-s)^2}$. You do NOT need to expand nor simplify your final quadratic polynomial.

5. Repeat problem 4 for hyperbola.

6. (Extra Credit - 15 points) Write down the equation for parabola with focus (p, q) and directrix $y = ax+b$. Again, you do NOT need to expand nor simplify your final quadratic polynomial.