

Name and Student ID: _____

Homework 4, Analytic Geometry and Matrices

Problems concerning conics:

1. Prove the statement claimed in class: every point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b > 0$, satisfies

$$\overline{PF} = e\overline{PD}.$$

Here, e is the eccentricity, F and D is a set of focus and directrix on the same side.

2. Find the eccentricities, foci, and directrices of the following conic sections:

(a) $6x^2 + 9y^2 = 54$

(b) $9x^2 + 36x - 16y^2 = 126$

(c) $y^2 - 2x = 4$

3. Sketch the following curves, given as an image of Φ of an $r\theta$ equation. If the curve is a conic, specify foci and directrices:

(a) $r = \frac{4}{2-2\cos\theta}$

(b) $r = \frac{12}{3+3\sin\theta}$

(c) $r = \frac{25}{10-5\cos(\theta-\frac{\pi}{3})}$

(d) $r \cos(\theta + \frac{\pi}{6}) = 2$

Problems concerning quadric surfaces:

1. Sketch the traces of the following equations on \mathbb{R}^3 . Be sure to label intersections with all coordinate axes:

(a) $x^2 = 1 - y - z^2$

(b) $y^2 - x^2 = z$

(c) $16y^2 + 9z^2 = 4x^2$

2. Which of the traces above is a graph of a function? Be sure to provide the corresponding function.