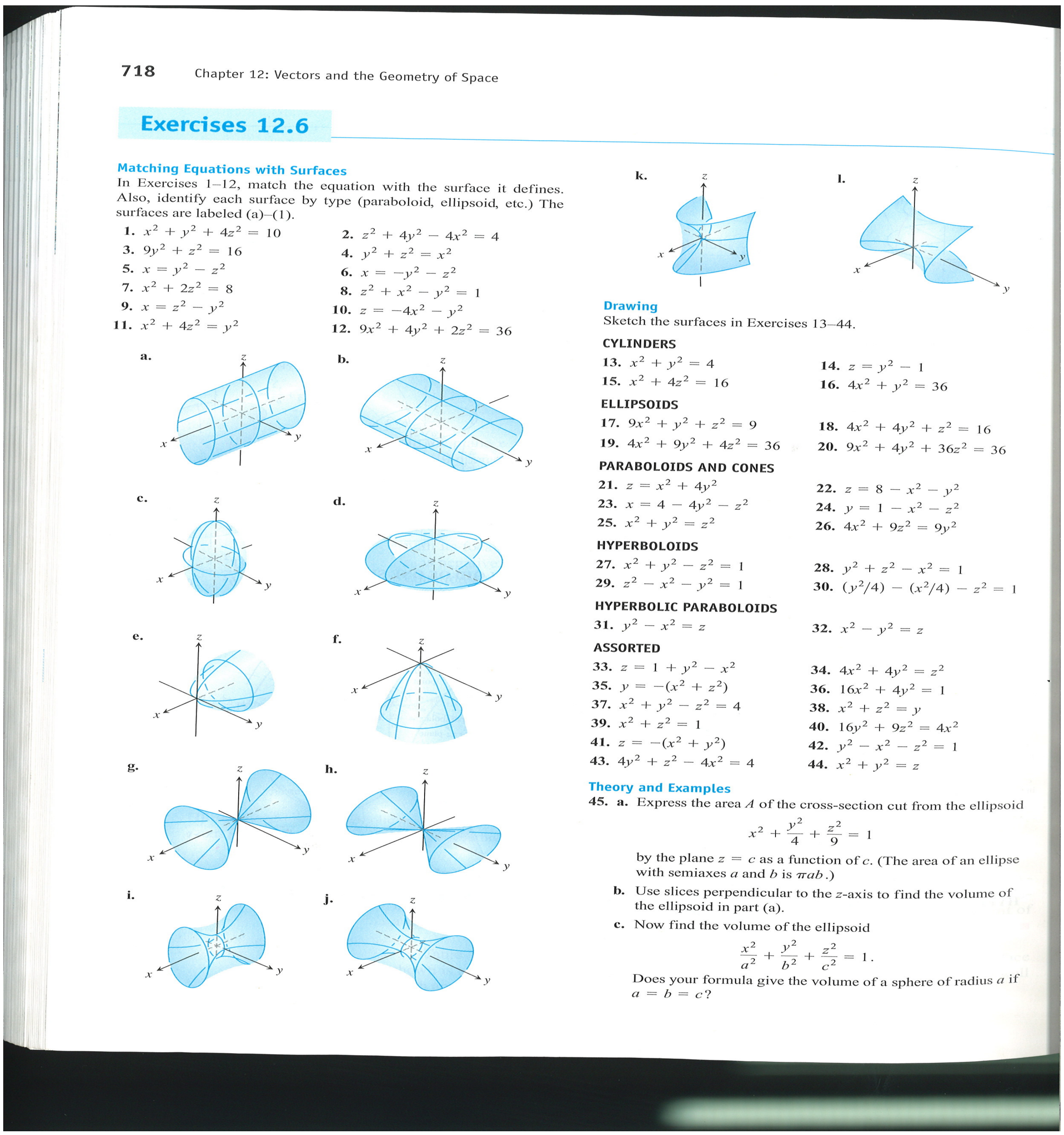
Name and Student ID: ______________________________

Homework 5, Analytic Geometry and Matrices

Determination of Traces:

1. Each defining equation below corresponds to only one possible trace from sketches a through l below. Match it and provide sufficient reasons.
   (a) $x = 2z^2 - 2y^2$.
   (b) $9x^2 + 4y^2 + 2z^2 = 36$.
   (c) $x = -y^2 - z^2$.
   (d) $z = -2x^2 - y^2$.



Exercises 12.6

Matching Equations with Surfaces
In Exercises 1–12, match the equation with the surface it defines. Also, identify each surface by type (paraboloid, ellipsoid, etc.) The surfaces are labeled (a)–(l).

1. $x^2 + y^2 + 4z^2 = 10$
2. $z^2 + 4y^2 - 4x^2 = 4$
3. $9y^2 + z^2 = 16$
4. $y^2 + z^2 = x^2$
5. $x = y^2 - z^2$
6. $x = -y^2 - z^2$
7. $x^2 + 2z^2 = 8$
8. $z^2 + x^2 - y^2 = 1$
9. $x = z^2 - y^2$
10. $z = -4x^2 - y^2$
11. $x^2 + 4z^2 = y^2$
12. $9x^2 + 4y^2 + 2z^2 = 36$

Drawing
Sketch the surfaces in Exercises 13–44.

CYLINDERS
13. $x^2 + y^2 = 4$
14. $z = y^2 - 1$
15. $x^2 + 4z^2 = 16$
16. $4x^2 + y^2 = 36$

ELLIPSOIDS
17. $9x^2 + y^2 + z^2 = 9$
18. $4x^2 + 4y^2 + z^2 = 16$
19. $4x^2 + 9y^2 + 4z^2 = 36$
20. $9x^2 + 4y^2 + 36z^2 = 36$

PARABOLOIDS AND CONES
21. $z = x^2 + 4y^2$
22. $z = 8 - x^2 - y^2$
23. $x = 4 - 4y^2 - z^2$
24. $y = 1 - x^2 - z^2$
25. $x^2 + y^2 = z^2$
26. $4x^2 + 9z^2 = 9y^2$

HYPERBOLOIDS
27. $x^2 + y^2 - z^2 = 1$
28. $y^2 + z^2 - x^2 = 1$
29. $z^2 - x^2 - y^2 = 1$
30. $(y^2/4) - (x^2/4) - z^2 = 1$

HYPERBOLIC PARABOLOIDS
31. $y^2 - x^2 = z$
32. $x^2 - y^2 = z$

ASSORTED
33. $z = 1 + y^2 - x^2$
34. $4x^2 + 4y^2 = z^2$
35. $y = -(x^2 + z^2)$
36. $16x^2 + 4y^2 = 1$
37. $x^2 + y^2 - z^2 = 4$
38. $x^2 + z^2 = y$
39. $x^2 + z^2 = 1$
40. $16y^2 + 9z^2 = 4x^2$
41. $z = -(x^2 + y^2)$
42. $y^2 - x^2 - z^2 = 1$
43. $4y^2 + z^2 - 4x^2 = 4$
44. $x^2 + y^2 = z$

Theory and Examples
45. **a.** Express the area A of the cross-section cut from the ellipsoid

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$$

by the plane $z = c$ as a function of c. (The area of an ellipse with semiaxes a and b is πab.)

b. Use slices perpendicular to the z-axis to find the volume of the ellipsoid in part (a).

c. Now find the volume of the ellipsoid

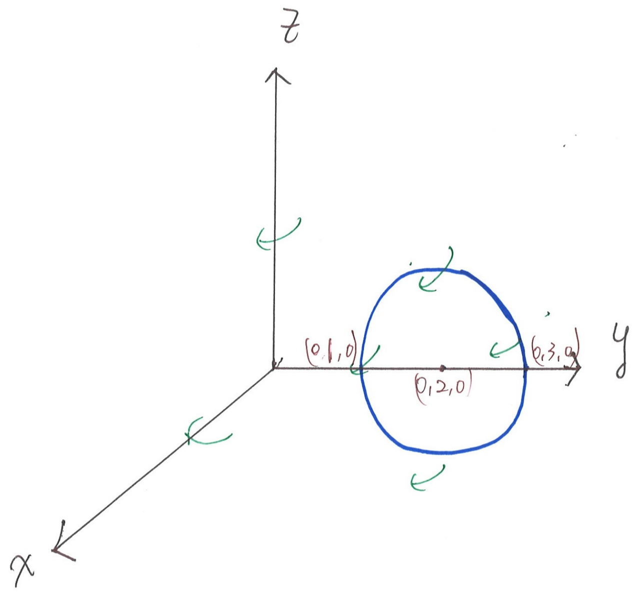
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Does your formula give the volume of a sphere of radius a if $a = b = c$?

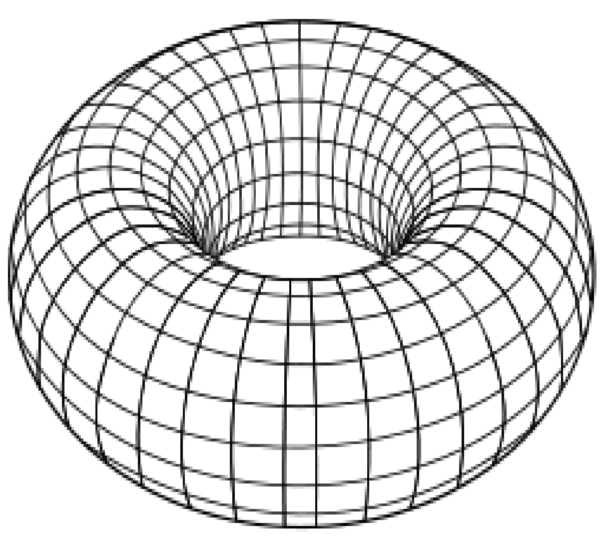
More Change of Coordinates:

1. Find appropriate change of coordinate that turns the regions below into regions of constant bounds (i.e. rectangles or boxes)
   (a) $9x^2 + 16y^2 + 4z^2 - 8z - 140 \leq 0$ and $z \geq 1$.
   (b) Region in $\mathbb{R}^2$ bounded by $y_1 = 3x+8$, $y_2 = -2x+7$, $y_3 = 3x+12$, and $y_4 = -2x+10$.

2. A 1-torus in $\mathbb{R}^3$ is formed by rotating the circle on yz plane with radius 1 and center $(0, 2, 0)$



around z-axis to form a "donut":



(a) Construct a change of coordinate that turns the torus into a square $[0, 2\pi) \times [0, 2\pi)$. (Hint: Rotate the circle on yz plane by θ, around z-axis and figure out another angle ϕ so that (θ, ϕ) determines the points on the torus.)

(b) Sketch the images of $\theta = \pi$ and $\phi = \frac{\pi}{2}$ on the torus under this change of coordinate.

(c) Construct a change of coordinate that turns the region bounded by torus into a box $[0, 2\pi) \times [0, 2\pi) \times [0, 1]$.

Vectors in $\mathbb{R}^3$:

1. Prove that two lines having two distinct points in common must be the same line. That is, $x \neq x'$ and $x, x' \in L \cap L' \Rightarrow L = L'$. (Use the definition of lines and conditions of equality stated in class.) Use this fact to prove that the line $(1, -1, 0) + \mathbb{R}(1, 1, 1)$ is the same as the line given by the intersection of two planes $x + y - 2z = 0$ and $2x - y - z = 3$.