Name and Student ID: ______________________________

Homework 6, Analytic Geometry and Matrices

Subspaces and Spans:

1. Determine, with sufficient reason, whether the following subset of $\mathbb{R}^3$ is a subspace.
   (a) $W = \{(x, y, z) \mid x = 3y \text{ and } z = -y\}$
   (b) $W = \{(x, y, z) \mid x = z + 2\}$
   (c) $W = \{(x, y, z) \mid 2x - y + z = 0\}$
   (d) $W = \{(x, y, z) \mid x + 6y - 9z = 1\}$
   (e) $W = \{(x, y, z) \mid x^2 + y = 0\}$

2. If U, V are subspaces of $(\mathbb{R}^n, +, \cdot)$, prove that
   (a) $U \cap V$ is a subspace.
   (b) $U + V := \{u + v \mid u \in U \text{ and } v \in V\}$ is a subspace.
   (c) $U \cup V$ is NOT necessarily a subspace. (Give an example that it fails to be a subspace).

3. Determine, for each part, if the first vector is in the span of the other two vectors.
   (a) $(-2, 0, 3)^T, (1, 3, 0)^T, (2, 4, 1)^T$.
   (b) $(3, 4, 1)^T, (1, -2, 1)^T, (-2, -1, 1)^T$.
   (c) $(-2, 2, 2)^T, (1, 2, -1)^T, (-3, -3, 3)^T$.

Plane Geometry

1. Cross product of two vectors $u, v \in \mathbb{R}^3$ satisfy $|u \times v| = |u||v| \sin\theta$, where θ is the angle between $\mathbb{R}u$ and $\mathbb{R}v$. That is, the $|u \times v|$ is the area of the parallelogram spanned by u and v. Prove this fact for vectors on xy plane. That is, when $u = (u_1, u_2, 0)^T$ and $v = (v_1, v_2, 0)^T$.

2. Prove that the distance from a point $(x_0, y_0, z_0)^T$ to the plane given by $ax+by+cz+d = 0$ is

$$\frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$