Name and Student ID: ______________________________

Homework 9, Analytic Geometry and Matrices

1. Continue from Problem 7 in Homework 8, consider the following map:

$$P : V \to V$$

defined by

$$T(v) = v_2.$$

This map is called the projection onto W_2 along W_1.

   (a) Check that T is a well-defined linear transformation.

   (b) Write down $[T]_\gamma^\gamma$, where γ is the basis given in in that problem.

   (c) Prove that $T^n := \underbrace{T \circ \cdots \circ T}_{\text{n times}} = T$ for all n.

2. Prove that $\mathcal{L}(V, W)$, with addition and scalar multiplication defined in class, is a vector space. (Need to check the 8 requirements V1-V8 for a vector space).

3. Let V and W be vector spaces, and let $S, T \in \mathcal{L}(V, W)$ be nonzero linear transformations between them. If $\mathcal{R}(S) \cap \mathcal{R}(T) = \{0\}$, prove that S and T are linear independent in the vector space $\mathcal{L}(V, W)$.

4. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & 3 \\ 1 & -2 & 1 \\ 1 & -3 & 1 \end{pmatrix}.$$

Find elementary operations that transform A into B, B into I_3, the identity matrix. Express (but do not actually multiply out) the matrix that transform A into I_3.

An $n \times n$ matrix $A = (a_{ij})$ is called *upper triangular* if all the entries below the diagonal are zero. That is, $a_{ij} = 0 \; \forall \; i > j$.

5. Prove that a product of two upper triangular matrices is upper triangular, and therefore a product of any number of upper triangular matrices is upper triangular. Remember the fact that when bases are fixed, matrices are in one-to-one correspondence with linear transformations.

6. Find elementary operation that transform

$$A = \begin{pmatrix} 2 & 2 & 4 \\ 4 & 0 & 1 \\ 6 & -1 & 4 \end{pmatrix}$$

into a upper triangular matrix.