Hw 10:

§5-5: 8, ~~10~~, 12, 24,

§4-9: 4, 26,

§5-6: 12, 24, 30, 36, 39,

§5-8: 7, 8, 9, 10, 26, 28, 37.

各5分

+5分完成度

§4-9:

※4. Find the position, velocity, and acceleration at time t_0. What is the speed at time t_0?

$$x(t) = \frac{2t}{t+3}, \quad t_0 = 3$$

$$x(3) = \frac{2\times 3}{3+3} = 1$$

$$v(t) = x'(t) = \frac{2(t+3) - 2t}{(t+3)^2} = \frac{6}{(t+3)^2}$$

$$x'(3) = \frac{6}{36} = \frac{1}{6} = \text{speed}$$

$$a(t) = x''(t) = \frac{0 - 6\cdot 2\cdot(t+3)}{(t+3)^4} = \frac{-12}{(t+3)^3}$$

$$x''(3) = \frac{-12}{216} = \frac{-1}{18}$$

※26. Determine the time intervals, if any, during which the object satisfies the given condition.

$x(t) = t^3 - 6t^2 - 15t$, moves right slowing down.

, that is, $v(t) > 0$ and $a(t) < 0$

$$v(t) = 3t^2 - 12t - 15 = 3(t-5)(t+1)$$

$$a(t) = 6t - 12 = 6(t-2)$$

t		-1		2		5	
$v(t)$:	+		−		−		+
$a(t)$:	−		−		+		+

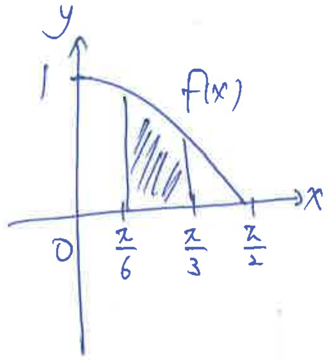
$\Rightarrow$ This never happens.

§5-5

· Find area

※8

$f(x) = \cos x, \quad x \in [\frac{\pi}{6}, \frac{\pi}{3}]$

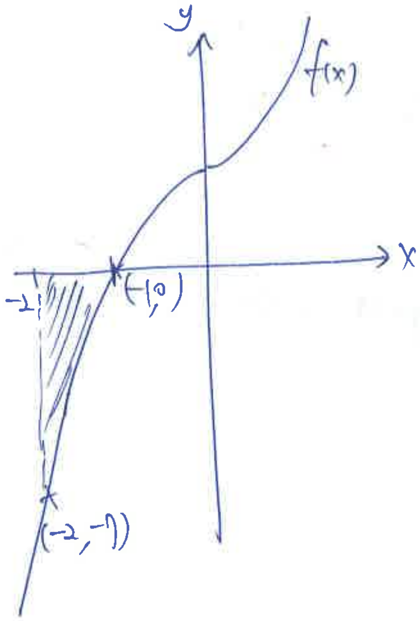


$$\text{area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x \, dx = \sin x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \sin\frac{\pi}{3} - \sin\frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}$$

※10.

$f(x) = x^3 + 1, \quad x \in [-2, -1]$



$$\text{area} = -\int_{-2}^{-1} x^3 + 1 \, dx$$

$$= -\left(\frac{1}{4}x^4 + x\right)\Big|_{-2}^{-1}$$

$$= \frac{3}{4} - (-2) = \frac{11}{4}$$

§5-5

Sketch the region bounded by the curves and find its area.

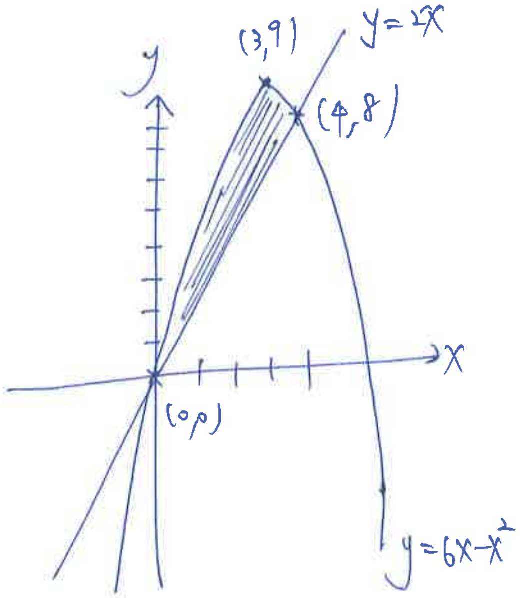
※12. $y = 6x - x^2$, $y = 2x$

$= -(x-3)^2 + 9$

$\begin{cases} y = 6x - x^2 \\ y = 2x \end{cases} \Rightarrow 2x = 6x - x^2$

$x^2 - 4x = 0$

$x = 0, 4$



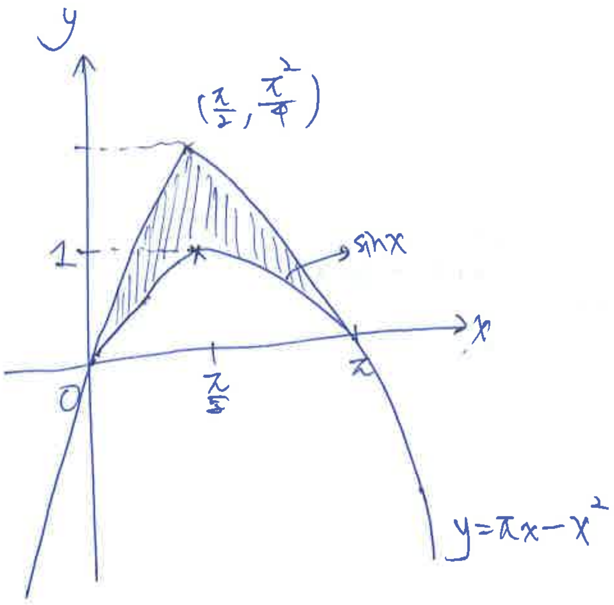
$$\text{area} = \int_0^4 (6x - x^2) - (2x)\, dx$$

$$= \int_0^4 4x - x^2\, dx$$

$$= 2x^2 - \frac{1}{3}x^3 \Big|_0^4 = 32 - \frac{64}{3} = \frac{32}{3}$$

※24. $y = \sin x$, $y = \pi x - x^2$

$= -(x - \frac{\pi}{2})^2 + \frac{\pi^2}{4}$



$$\text{area} = \int_0^\pi (\pi x - x^2) - (\sin x)\, dx$$

$$= \int_0^\pi \pi x - x^2 - \sin x\, dx$$

$$= \frac{\pi}{2}x^2 - \frac{1}{3}x^3 + \cos x \Big|_0^\pi$$

$$= \left[\frac{\pi^3}{2} - \frac{\pi^3}{3} + (-1)\right] - \left[0 - 0 + 1\right]$$

$$= \frac{\pi^3}{6} - 2$$

§5-6.

※12, Calculus:

$$\int (2-\sqrt{x})(2+\sqrt{x})\,dx$$

$$= \int 4-x\,dx$$

$$= 4x - \frac{1}{2}x^2 + C$$

※24,

$f'(x) = \cos x$, $f(\pi)=3$

$f(x) = \int \cos x\,dx = \sin x + C$

$f(\pi) = \sin\pi + C = 3$

$C = 3$

$\therefore f(x) = \sin x + 3$

※30,

$f''(x) = \sin x$, $f'(0) = -2$, $f(0) = 1$

$f'(x) = \int \sin x\,dx = -\cos x + C$

$f'(0) = -\cos 0 + C = -1 + C = -2$, $C = -1$, $f'(x) = -\cos x - 1$

$f(x) = \int -\cos x - 1\,dx = -\sin x - x + C$

$f(0) = -\sin 0 - 0 + C = C = 1$

$\therefore f(x) = -\sin x - x + 1$

§5-6

※36.

acceleration $a(t) = (t+2)^3$, $t \geq 0$

velocity: $V(t) = \int (t+2)^3 dx = \int u^3 du = \frac{1}{4}u^4 + C = \frac{1}{4}(t+2)^4 + C$

$u = t+2$
$du = dt$

$V(0) = \frac{1}{4} \times 16 + C = 4 + C = 3$, $C = -1$

$V(t) = \frac{1}{4}(t+2)^4 - 1$

position: $x(t) = \int \frac{1}{4}(t+2)^4 - 1 \, dx = \int \frac{1}{4}u^4 - 1 \, du = \frac{1}{20}u^5 - u + C$

$= \frac{1}{20} \cdot (t+2)^5 - (t+2) + C$

$u = t+2$
$du = dt$

$x(0) = \frac{1}{20} \times 2^5 - 2 + C = \frac{32}{20} - 2 + C = \frac{-8}{20} + C = 0$, $C = \frac{8}{20} = \frac{2}{5}$

(∵ origion)

∴ $X(t) = \frac{1}{20}(t+2)^5 - (t+2) + \frac{2}{5}$

$= \frac{(t+2)^5}{20} - t - \frac{8}{5}$

39.

(a)

$V_0 = 60$ mph $= 88$ feet per second.

$a = -20$ feet per second.

$\therefore V(t) = at + V_0 = -20t + 88$

Let $V(t) = 0$, $-20t + 88 = 0 \Rightarrow t = 4.4$ seconds.

(b)

$X(t) = \int -20t + 88 \, dt = -10t^2 + 88t + X_0$

$X_0 = 0$

$\Rightarrow X(t) = -10t^2 + 88t$

$X(4.4) = -10 \times 4.4^2 + 88 \times 4.4 = 193.6$ feet

§5-8

assume $\int_a^b f(x)\,dx = 0$, f: continuous on $[a,b]$

※7. No

$f(x)$ = odd function defined on $[-c, c]$

$f(x) = x$ defined on $[-1,1]$

$$\int_{-1}^{1} x\,dx = \frac{1}{2}x^2\Big|_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$$

※8. Yes if $f(x) \neq 0$, for each $x \in [a,b]$

then by continuity either $f(x) > 0$ for all $x \in [a,b]$

or $f(x) < 0$ for all $x \in [a,b]$.

In either case

$$\int_a^b f(x)dx \neq 0.$$

※9. No,

$$\int_{-1}^{1} x\,dx = 0, \text{ but } \int_{-1}^{1} |x| = \int_{-1}^{0} -x\,dx + \int_0^1 x\,dx$$

$$= \frac{-1}{2}x^2\Big|_{-1}^{0} + \frac{1}{2}x^2\Big|_0^1$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

$$\neq 0$$

※10. Yes

$$\because \int_a^b f(x)\,dx = 0$$

$$\Rightarrow \left| \int_a^b f(x)\,dx \right| = |0| = 0$$

Calculate:

※26.

$$\frac{d}{dx}\left(\int_{\sqrt{x}}^{x^2+x} \frac{1}{2+\sqrt{t}}\,dt \right)$$

$$= \frac{d}{dx}\left(\int_{\sqrt{x}}^{a} \frac{1}{2+\sqrt{t}}\,dt + \int_a^{x^2+x} \frac{1}{2+\sqrt{t}}\,dt \right) \qquad a = 常數$$

$$= \frac{d}{dx}\left(-\int_a^{\sqrt{x}} \frac{1}{2+\sqrt{t}}\,dt \right) + \frac{d}{dx}\left(\int_a^{x^2+x} \frac{1}{2+\sqrt{t}}\,dt \right)$$

$$= -\frac{1}{2+\sqrt[4]{x}} \cdot (\sqrt{x})' + \frac{1}{2+\sqrt{x^2+x}} \cdot (x^2+x)'$$

$$= \frac{-1}{2\sqrt{x}} \cdot \frac{1}{2+\sqrt[4]{x}} - \frac{2x+1}{2+\sqrt{x^2+x}}$$

§5-8

※28.

$$\frac{d}{dx}\left(\int_{3x}^{1/x} \cos(2t)\,dt\right)$$

$$= \frac{d}{dx}\int_{3x}^{a} \cos(2t)\,dt + \frac{d}{dx}\int_{a}^{1/x} \cos(2t)\,dt \quad , \quad a:常數$$

$$= -\frac{d}{dx}\int_{a}^{3x} \cos(2t)\,dt + \cos\left(\frac{2}{x}\right)\cdot\left(\frac{1}{x}\right)'$$

$$= -\cos(6x)\cdot(3x)' + \cos\left(\frac{2}{x}\right)\cdot\left(\frac{-1}{x^2}\right)$$

$$= -3\cos(6x) - \cos\left(\frac{2}{x}\right)\cdot\frac{1}{x^2}$$

※37. Evaluate:

$$\int_{-\pi/3}^{\pi/3} (1+x^2-\cos x)\,dx \qquad \because\ 1+x^2-\cos x\ \text{為 even function}$$

$$= 2\times\int_{0}^{\frac{\pi}{3}} (1+x^2-\cos x)\,dx$$

$$= 2\times\left[x+\frac{1}{3}x^3-\sin x\right]\Big|_0^{\frac{\pi}{3}}$$

$$= 2\times\left(\frac{\pi}{3}+\frac{1}{3}\times\frac{\pi^3}{27}-\sin\frac{\pi}{3}\right)$$

$$= \left(\frac{\pi}{3}+\frac{\pi}{81}-\frac{\sqrt{3}}{2}\right)\times 2 = \frac{2\pi}{3}+\frac{2\pi^3}{81}-\sqrt{3}$$