Hw11:

§5-9: 12, 14, 22

§6-1: 6, 12, 22, 36,

§6-2: 6, 14, 16, 24, 34, 40.

§5-9

※12

$f(x) = \cos x$, $x \in [0, \pi]$

average value $= \frac{1}{\pi}\int_0^{\pi} \cos x \, dx = \frac{1}{\pi}\left[\sin x \Big|_0^{\pi}\right] = 0$

$f(x) = \cos x = 0 \Rightarrow x = \frac{\pi}{2}$

※14. Given that f is continuous on $[a,b]$, compare $f(b)(b-a)$ and $\int_a^b f(x)\,dx$

(a) if f is constant on $[a,b]$

(b) if f increases on $[a,b]$

(c) if f decreases on $[a,b]$

average value $= \frac{1}{b-a} \cdot \int_a^b f(x)\,dx = f(c)$, for some point $x = c$, $a < c < b$,

$\Rightarrow \int_a^b f(x)\,dx = f(c)(b-a)$

(a)

f is constant on $[a,b] \Rightarrow f(c) = f(b) \Rightarrow \int_a^b f(x)\,dx = f(b)(b-a)$

(b)

f increases on $[a,b] \Rightarrow f(c) < f(b) \Rightarrow \int_a^b f(x)\,dx < f(b)(b-a)$

(c)

f decreases on $[a,b] \Rightarrow f(c) > f(b) \Rightarrow \int_a^b f(x)\,dx > f(b)(b-a)$

※ 22.

Show that the average value of the functions $f(x) = \sin(\pi x)$ and $g(x) = \cos(\pi x)$ is zero on every interval of length $2n$, n a positive integer.

<pf>

$f(x) = \sin(\pi x)$ on $[a, a+2n]$, $a \in \mathbb{R}$ and $g(x) = \cos(\pi x)$ on $[a, a+2n]$, $a \in \mathbb{R}$.

$$\text{average value} = \frac{1}{(a+2n)-a} \cdot \int_a^{a+2n} \sin(\pi x)\,dx$$

$$= \frac{1}{2n} \cdot \left[-\frac{1}{\pi} \cos(\pi x) \Big|_a^{a+2n} \right]$$

$$= \frac{1}{2n} \cdot \left[-\frac{1}{\pi} \underbrace{\cos(a\pi + 2n\pi)}_{=\cos(a\pi)} + \frac{1}{\pi} \cdot \cos(a\pi) \right] = 0$$

$$\text{average value} = \frac{1}{(a+2n)-a} \cdot \int_a^{a+2n} \cos(\pi x)\,dx$$

$$= \frac{1}{2n} \cdot \left[\frac{1}{\pi} \cdot \sin(\pi x) \Big|_a^{a+2n} \right]$$

$$= \frac{1}{2n} \cdot \left[\frac{1}{\pi} \cdot \underbrace{\sin(a\pi + 2n\pi)}_{=\sin(a\pi)} - \frac{1}{\pi} \cdot \sin(a\pi) \right] = 0$$

§ 6-1

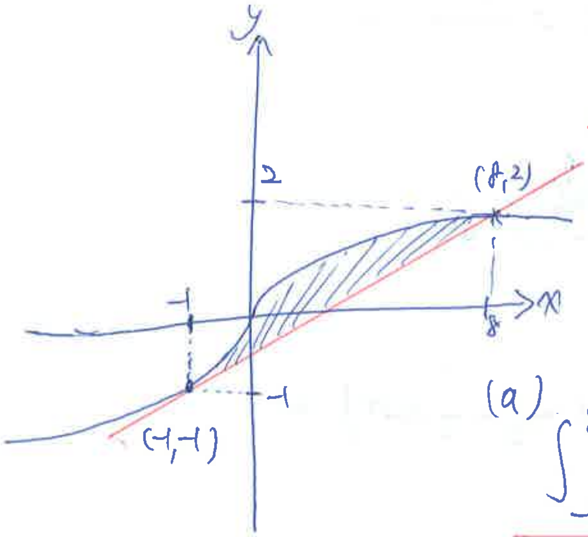
#6. $x=y^3$, $x=3y+2$

$\begin{cases} x=y^3 \\ x=3y+2 \end{cases} \Rightarrow y^3=3y+2 \Rightarrow y^3-3y-2=0$

$\Rightarrow (y+1)(y^2-y-2)=0$

$\Rightarrow (y+1)(y+1)(y-2)=0$

$\Rightarrow \begin{cases} y=2 \\ x=8 \end{cases} \quad \begin{cases} y=-1 \\ x=-1 \end{cases}$



$x=3y+2$, $y=\frac{x-2}{3}$

$x=y^3$, $y=x^{\frac{1}{3}}$

(a) $\int_{-1}^{8} x^{\frac{1}{3}} - \left(\frac{x-2}{3}\right) dx$

(b) $\int_{-1}^{2} (3y+2) - y^3 \, dy$

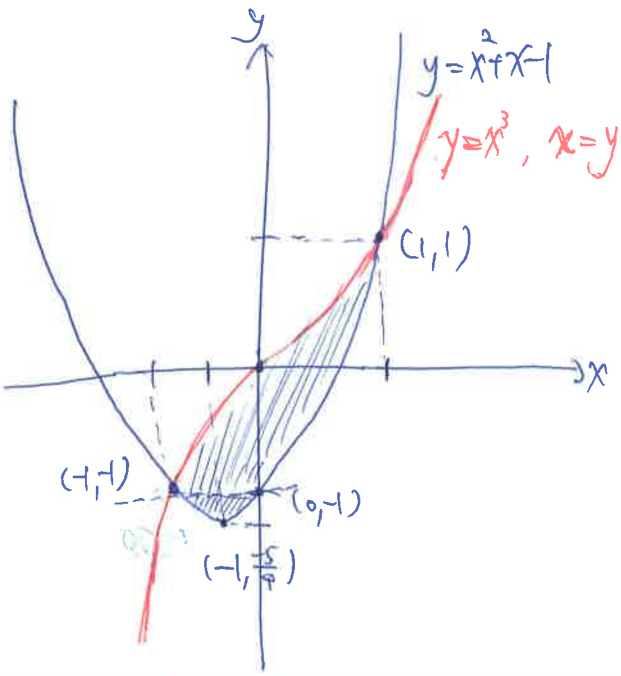
#12,

$y=x^3$, $y=x^2+x-1 = \left(x+\frac{1}{2}\right)^2 - \frac{5}{4} \Rightarrow \left(x+\frac{1}{2}\right)^2 = y+\frac{5}{4} \Rightarrow x=\frac{-1}{2} \pm \sqrt{y+\frac{5}{4}}$

$\begin{cases} y=x^3 \\ y=x^2+x-1 \end{cases} \Rightarrow x^3=x^2+x-1 \Rightarrow x^3-x^2-x+1=0 \Rightarrow (x-1)(x^2-1)=0 \Rightarrow (x-1)^2(x+1)=0$

$\Rightarrow x^2(x-1)-(x-1)=0$

$\Rightarrow \begin{cases} x=1 \\ y=1 \end{cases} \quad \begin{cases} x=-1 \\ y=-1 \end{cases}$



$y=x^3$, $x=y^{1/3}$

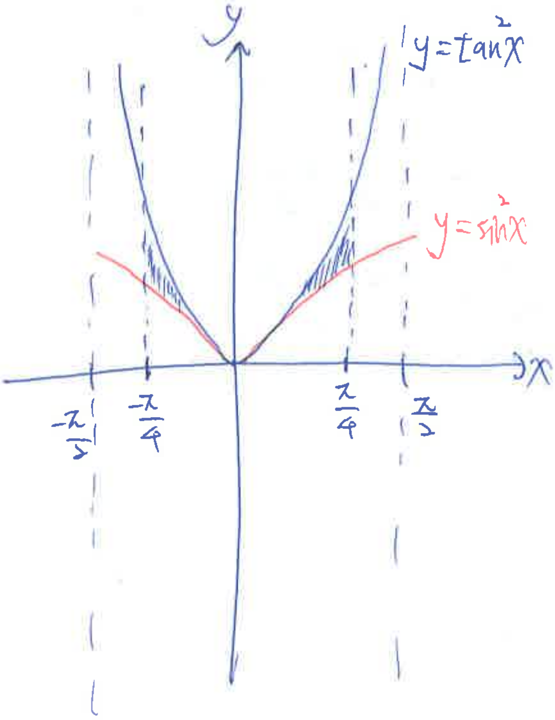
(a) $\int_{-1}^{1} x^3 - (x^2+x-1) \, dx$

(b) $\int_{-\frac{5}{4}}^{-1} \left[\frac{-1}{2}+\frac{1}{2}\sqrt{4y+5}\right] - \left[\frac{-1}{2}-\frac{1}{2}\sqrt{4y+5}\right] dy$

$+ \int_{-1}^{1} \left[\frac{-1}{2}+\frac{1}{2}\sqrt{4y+5}\right] - y^{1/3} \, dy$

※ 22.

$y = \sin^2 x$, $y = \tan^2 x$, $x \in [\frac{-\pi}{4}, \frac{\pi}{4}]$, use . $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$



$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} \geqslant \frac{\sin^2 x}{1} = \sin^2 x$$

$$A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x - \sin^2 x \, dx$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sec^2 x - 1) - \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x + \frac{1}{2}\cos 2x - \frac{3}{2} \, dx$$

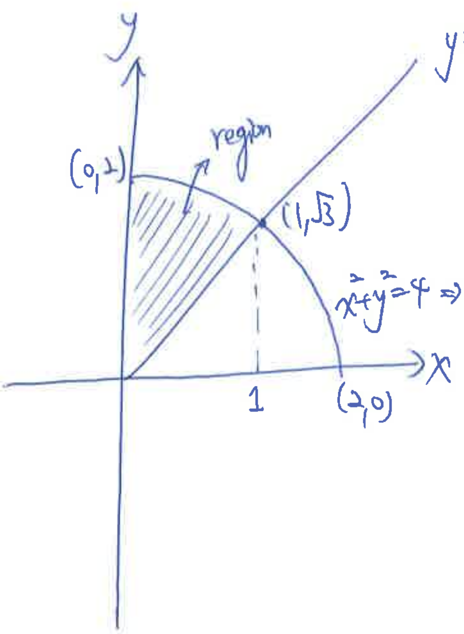
$$= \tan x + \frac{1}{4} \cdot \sin 2x - \frac{3}{2}x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \frac{5}{2} - \frac{3\pi}{4}$$

※

§ 6-1

※36. The region in the first quadrant bounded by the y-axis, the line $y=\sqrt{3}x$, and the circle $x^2+y^2=4$



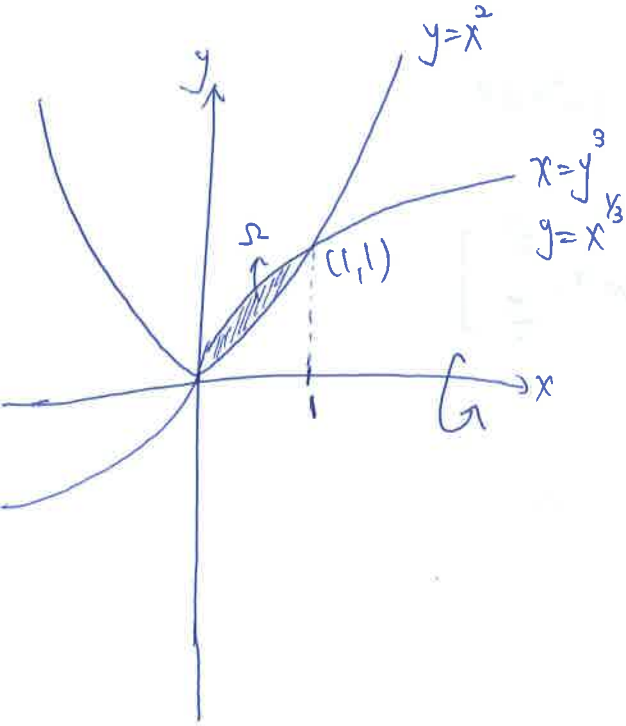
$y=\sqrt{3}x$ 代入 $\Rightarrow x^2+3x^2=4 \Rightarrow x=\pm 1$

$x^2+y^2=4 \Rightarrow y=\sqrt{4-x^2}$

$$A=\int_0^1 \sqrt{4-x^2}-\sqrt{3}x\,dx$$

§ 6-2

※6. $y=x^2$, $y=x^{1/3} \Rightarrow x=y^3$



$$V=\int_0^1 \pi(x^{1/3})^2-\pi\cdot(x^2)^2\,dx$$

$$=\int_0^1 \pi\cdot x^{\frac{2}{3}}-\pi\cdot x^4\,dx$$

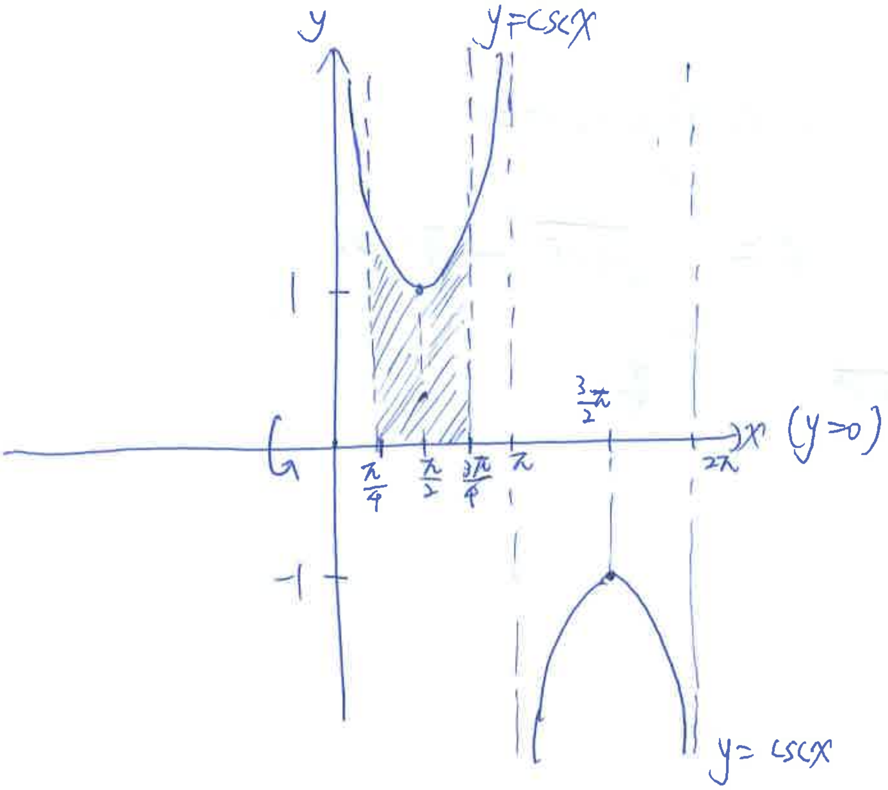
$$=\pi\cdot\int_0^1 x^{\frac{2}{3}}-x^4\,dx$$

$$=\pi\cdot\left[\frac{3}{5}\cdot x^{\frac{5}{3}}-\frac{1}{5}x^5\Big|_0^1\right]$$

$$=\frac{2\pi}{5}$$

14.

$y = \csc x$, $x = \frac{1}{4}\pi$, $x = \frac{3\pi}{4}$, $y = 0$



$$V = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \pi \cdot (\csc x)^2 - \pi \cdot 0^2 \, dx = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \pi \cdot \csc^2 x \, dx$$

$$= \pi \cdot \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \csc^2 x \, dx$$

$$= \pi \cdot \left[-\cot x \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \right]$$

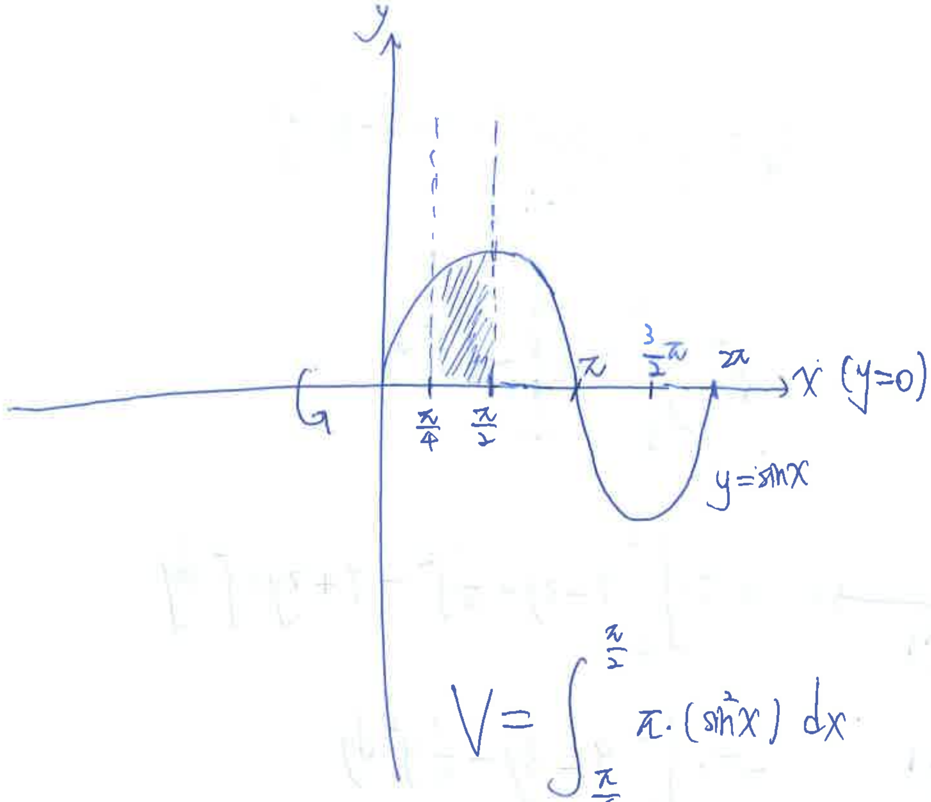
$$= 2\pi$$

#

§6-2

#16.

$y = \sin x$, $x = \frac{1}{4}\pi$, $x = \frac{1}{2}\pi$, $y = 0$,



$$V = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \pi \cdot (\sin^2 x)\, dx$$

$$= \pi \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\, dx$$

$$= \pi \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} - \frac{1}{2} \cdot \cos 2x\, dx$$

$$= \pi \cdot \left[\frac{1}{2}x - \frac{1}{4}\sin 2x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} \right]$$

$$= \frac{1}{8}\pi \cdot (\pi + 2)$$

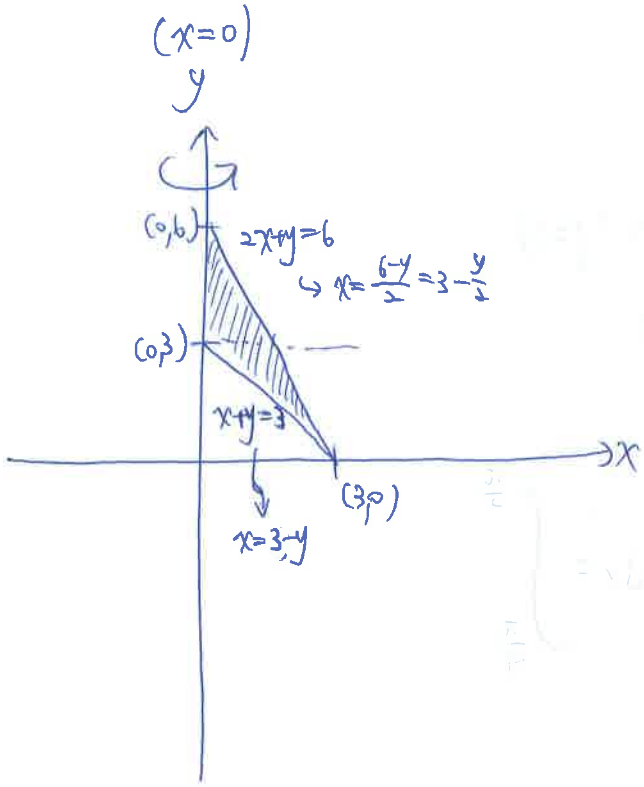
#

※ 24、

$x+y=3$, $2x+y=6$, $x=0$,

x	0	3
y	3	0

x	0	3
y	6	0



$$V=\int_0^3 \pi\cdot\left(3-\frac{y}{2}\right)^2-\pi\cdot(3-y)^2\,dy$$

$$+\int_3^6 \pi\cdot\left(3-\frac{y}{2}\right)^2 dy$$

$$=\pi\cdot\int_6^3 \cancel{9}-3y+\frac{1}{4}y^2-\cancel{9}+6y-y^2\,dy$$

$$+\pi\cdot\int_3^6 9-3y+\frac{1}{4}y^2\,dy$$

$$=\pi\cdot\int_0^3 3y-\frac{3}{4}y^2\,dy+\pi\cdot\int_3^6 9-3y+\frac{1}{4}y^2\,dy$$

$$=\pi\cdot\left[\frac{3}{2}y^2-\frac{1}{4}y^3\Big|_0^3\right]+\pi\cdot\left[9y-\frac{3}{2}y^2+\frac{1}{12}y^3\Big|_3^6\right]$$

$$=\pi\cdot\left(\frac{27}{2}-\frac{27}{4}\right)+\pi\cdot\left(\cancel{54}-\cancel{54}+\frac{\overset{18}{\cancel{216}}}{\cancel{12}}\right)-\pi\cdot\left[27-\frac{27}{2}+\frac{\overset{9}{\cancel{27}}}{\underset{4}{\cancel{12}}}\right]$$

$$=\pi\cdot\frac{27}{4}+18\pi-\pi\cdot\frac{63}{4}$$

$$=9\pi$$

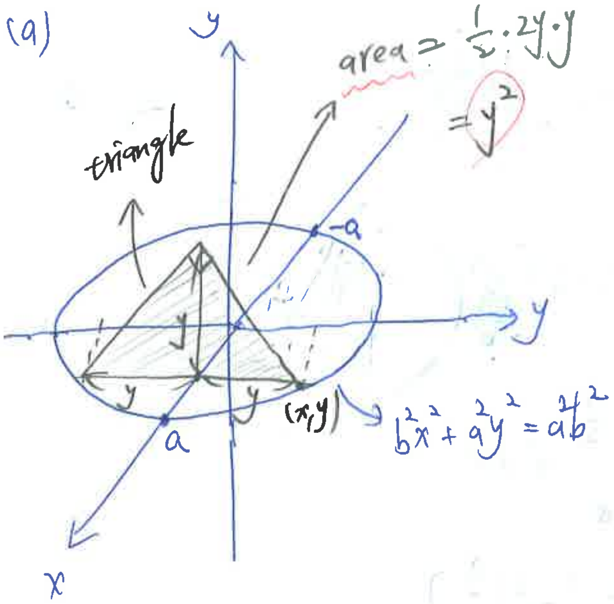
※

§6-2

※34.

$$b^2x^2 + a^2y^2 = a^2b^2 \Rightarrow y^2 = b^2 - \frac{b^2}{a^2}x^2$$

(a)



area of each triangle $= y^2$

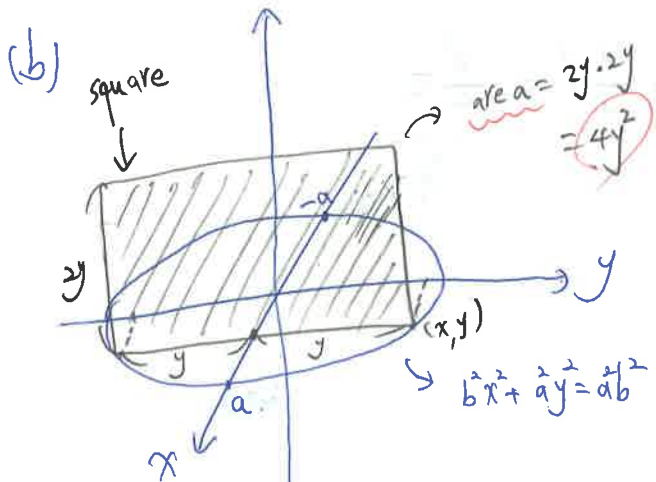
$$V = 2\cdot\int_0^a b^2 - \frac{b^2}{a^2}x^2\,dx$$

$$= 2\cdot\left[\, b^2x - \frac{b^2}{3a^2}x^3 \Big|_0^a \right]$$

$$= 2\cdot\left(ab^2 - \frac{1}{3}ab^2\right) = 2\cdot\frac{2}{3}ab^2 = \frac{4}{3}ab^2$$

※

(b)



area of each square $= 4y^2$

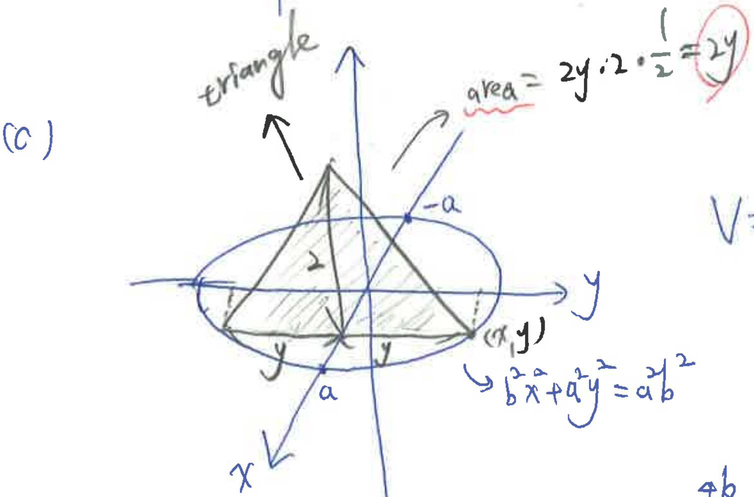
$$V = 2\cdot\int_0^a 4\left(b^2 - \frac{b^2}{a^2}x^2\right)dx$$

$$= 4\cdot\left[2\cdot\int_0^a b^2 - \frac{b^2}{a^2}x^2\,dx\right]$$

$$= 4\times\frac{4}{3}ab^2 = \frac{16}{3}ab^2$$

※

(c)



area of each triangle $= 2y$

$$V = 2\cdot\int_0^a 2\cdot\sqrt{b^2 - \frac{b^2}{a^2}x^2}\,dx$$

$$= \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= \frac{4b}{a}\cdot\left[\frac{x}{2}\cdot\sqrt{a^2-x^2} + \frac{a^2}{2}\cdot\sin^{-1}\left(\frac{x}{a}\right)\Big|_0^a\right]$$

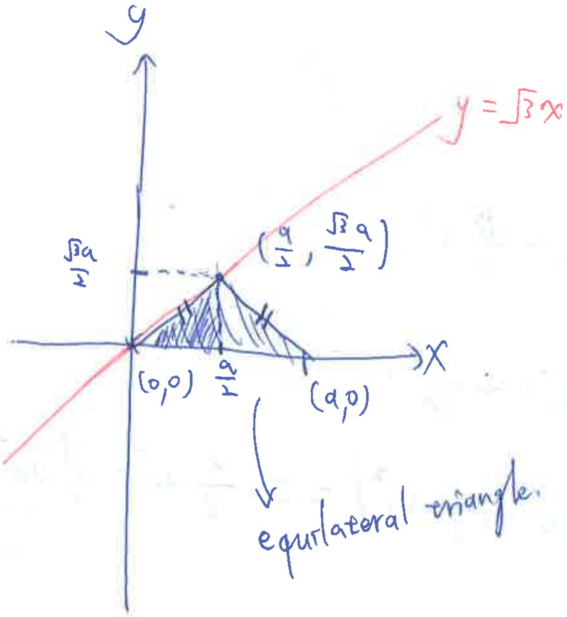
$$= \pi ab.$$

※

$\because \int\sqrt{a^2-x^2}\,dx$

$= \frac{x}{2}\cdot\sqrt{a^2-x^2} + \frac{a^2}{2}\cdot\sin^{-1}\left(\frac{x}{a}\right) + C$

※ 40.



$$V = 2 \times \int_0^{\frac{a}{2}} \pi \cdot (\sqrt{3}x)^2 dx$$

$$= 2 \times \int_0^{\frac{a}{2}} \pi \cdot 3x^2 dx$$

$$= 6\pi \cdot \int_0^{\frac{a}{2}} x^2 dx$$

$$= 6\pi \cdot \left[\frac{1}{3}x^3 \Big|_0^{\frac{a}{2}} \right]$$

$$= 6\pi \cdot \frac{1}{3} \cdot \left(\frac{a}{2}\right)^3 = \frac{a^3}{4}\pi$$

※