Hw12

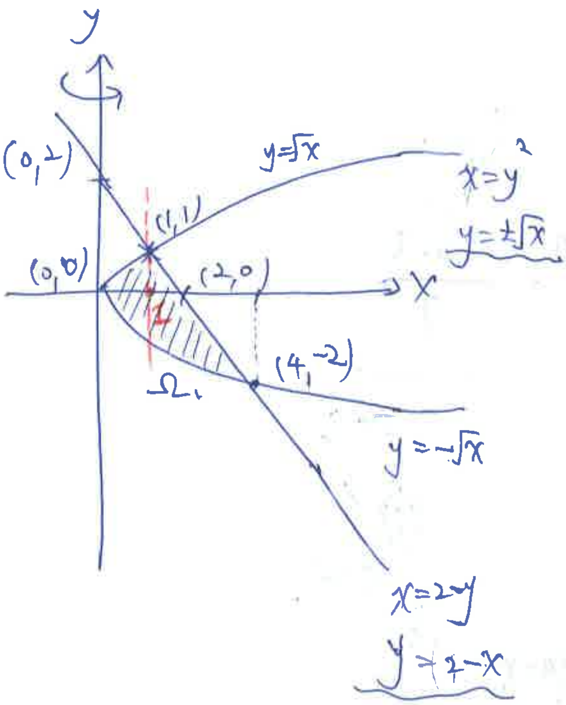
§6-3 : ※ 10、16、22、6

§7-1 : ※ 12、44、52、6、18、36、

§6-3

※10.

$x=y^2$, $x=2-y$ (shell method)



$$V=\int_0^1 2\pi x\cdot[\sqrt{x}-(-\sqrt{x})]\,dx+\int_1^4 2\pi x\,[(2-x)-(-\sqrt{x})]\,dx$$

$$=\int_0^1 2\pi x\cdot 2\sqrt{x}\,dx+\int_1^4 2\pi x\cdot[2-x+\sqrt{x}]\,dx$$

$$=\int_0^1 4\cdot\pi\cdot x^{\frac{3}{2}}\,dx+\int_1^4 4\pi x-2\pi x^2+2\pi x^{\frac{3}{2}}\,dx$$

$$=4\pi\cdot\frac{2}{5}\cdot x^{\frac{5}{2}}\Big|_0^1+4\pi\cdot\frac{1}{2}x^2-\frac{2\pi}{3}x^3+\frac{4\pi}{5}\cdot x^{\frac{5}{2}}\Big|_1^4$$

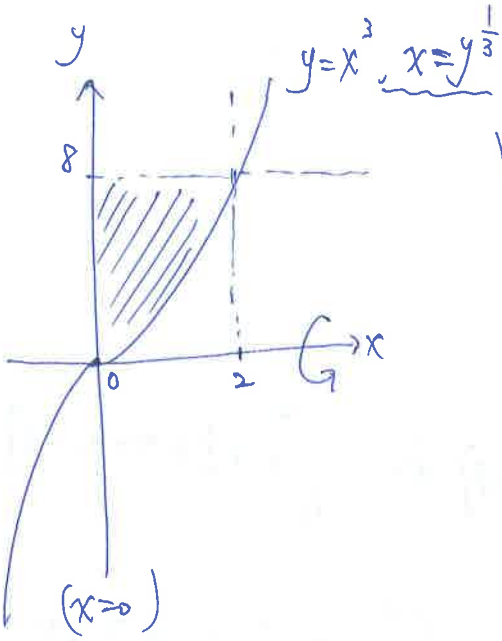
$$=\frac{8\pi}{5}+\left(32\pi-\frac{128\pi}{3}+\frac{128\pi}{5}\right)-\left(2\pi-\frac{2\pi}{3}+\frac{4\pi}{5}\right)$$

$$=\frac{72\pi}{5}$$ ※

§6-3

※16.

$y=x^3$, $y=8$, $x=0$, (shell method)



$$V=\int_0^8 2\pi y\cdot y^{\frac{1}{3}}\,dy$$

$$=\int_0^8 2\pi\cdot y^{\frac{4}{3}}\,dy$$

$$=2\pi\cdot\frac{3}{7}\cdot y^{\frac{7}{3}}\Big|_0^8$$

$$=\frac{6\pi}{7}\cdot 8^{\frac{7}{3}}=\frac{6\pi}{7}\cdot 2^7=\frac{768\pi}{7}$$

§7-1

※12.

① $f(x)=(4x-1)^3$

Let $f(x_1)=f(x_2)$

$(4x_1-1)^3=(4x_2-1)^3$

$4x_1-1=4x_2-1$

$4x_1=4x_2$

$x_1=x_2$

So, $f(x)$ is one-to-one

② Let $y=(4x-1)^3$

$y^{\frac{1}{3}}=4x-1$

$4x=1+\sqrt[3]{y}$

$x=\frac{1}{4}+\frac{1}{4}\cdot\sqrt[3]{y}$

So, $f^{-1}(x)=\frac{1}{4}+\frac{1}{4}\cdot\sqrt[3]{x}$

$f^{-1}(x)=\frac{1+\sqrt[3]{x}}{4}$, $\operatorname{dom} f^{-1}=(-\infty,\infty)$

§7-1

※44.

$f(x) = x - \pi + \cos x,\quad 0 < x < 2\pi,\quad c = -1$

$f'(x) = 1 - \sin x \Rightarrow f'(x) \geq 0$ on $[0, \pi] = I.$

~~with~~ $f'(x) = 0$ for only one value on I, so $f(x)$ has an inverse.

$f(0) = 1 - \pi$

$\because f(\pi) = -1 = c \Rightarrow (f^{-1})'(-1) = \dfrac{1}{f'(\pi)} = \dfrac{1}{1} = 1$ ※

$f'(\pi) = 1$

※52.

$f(x) = \displaystyle\int_1^{2x} \sqrt{16 + t^4}\, dt$

(a)

$\because f'(x) = \sqrt{16 + (2x)^4} \cdot \dfrac{d\,2x}{dx} = 2\sqrt{16 + 16x^4} > 0$ for all x

$\therefore f(x)$ has an inverse. ※

(b) Let $f(x) = 0 \Rightarrow \displaystyle\int_1^{2x} \sqrt{16 + t^4}\, dt = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$, that is, $f(\frac{1}{2}) = 0$,

$(f^{-1})'(0) = \dfrac{1}{f'(\frac{1}{2})} = \dfrac{1}{2\sqrt{17}} = \dfrac{\sqrt{17}}{34}$ ※

$f'(\frac{1}{2}) = 2 \cdot \sqrt{16 + 16 \cdot (\frac{1}{2})^4} = 2\sqrt{16 + 1} = 2\sqrt{17}$

HW12 More Solutions.

§6-3 : 6, ~~16~~, ~~16~~, 22

§7-1 : 6, 18, 36

§7-2 : 20, 22

§7-3 : 6, 14, 20, 28, 30

6-3

(6) $y_1 = x^2$, $y_2 = x^{\frac{1}{3}}$

$x^2 = x^{\frac{1}{3}} \rightsquigarrow x^6 - x = 0$

$x = 0, 1$

$x = \frac{1}{2} \Rightarrow y_1(\frac{1}{2}) = \frac{1}{4}$

$y_2(\frac{1}{2}) = (\frac{1}{2})^{\frac{1}{3}} > \frac{1}{2} > \frac{1}{4}$

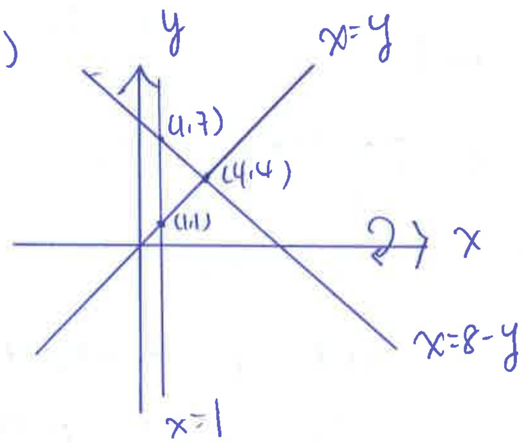
$\therefore y_2$ is the top curve.

$$V = 2\pi \int_0^1 (x^{\frac{1}{3}} - x^2)\, dx$$

$$= 2\pi \left(\frac{3}{4} x^{\frac{3}{4}} - \frac{x^3}{3} \right) \Big|_0^1$$

$$= \frac{5\pi}{6} \quad //$$

(22)



$$V = 2\pi \int_1^4 (y-1)\, dy + 2\pi \int_4^7 (8-y-1)\, dy$$

$$= 2\pi \int_1^4 (y-1)\, dy + 2\pi \int_4^7 (7-y)\, dy$$

$$= 2\pi \left(\frac{y^2}{2} - y \right) \Big|_1^4 + 2\pi \left(7y - \frac{y^2}{2} \right) \Big|_4^7$$

$$= 9\pi + \frac{9\pi}{2} = \frac{27}{2}\pi \quad //$$

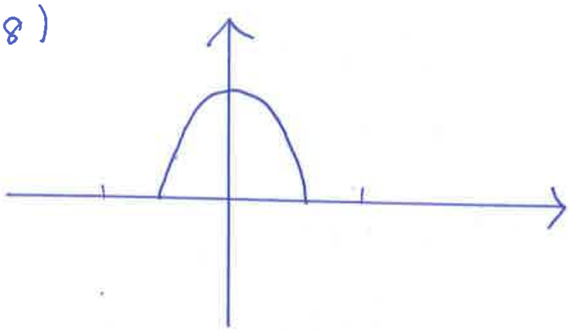
§7-1 p2

(6) $f(x) = x^2 - 3x + 2$

$f'(x) = 2x - 3 = 0$ at $\frac{3}{2}$ $\therefore$ f is not 1-1.

(a 1-1 function can't have zero derivative)

(18)



$\cos x : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [0, 1]$

is not 1-1 since

$\cos(-\frac{\pi}{2}) = \cos(\frac{\pi}{2}) = 0.$

(36) Note that $\frac{d}{dx}(1 - 2x - x^3) = -2 - 3x^2 < 0 \Rightarrow f$ decreasing $\therefore$ 1-1

We then solve $1 - 2x - x^3 = 4 \Rightarrow x^3 + 2x + 3 = 0$

$x = -1$ solves ~~and we divide the polynomials~~

$$\cancel{\begin{array}{r} x^2 - x \\ x+1 \overline{) x^3 + 0x^2 + 2x + 3} \\ x^3 + x^2 \\ \hline -x^2 + 2x \\ -x^2 \end{array}}$$

// possible roots $\frac{\pm 1}{\pm 3}$, $\frac{\pm 3}{\pm 1}$, ± 1.

and -1 happens to solve the poly. //

$$(f^{-1})'(4) = \frac{1}{f'(1)} = \frac{1}{-2-3} = -\frac{1}{5} \quad //$$

(20) $\frac{1}{2}\ln x = \ln(2x-1)$

$$\frac{1}{2}\ln x = \ln x^{\frac{1}{2}}$$

$\ln$ is 1-1 $\Rightarrow$ $\sqrt{x} = 2x-1$

$\Rightarrow$ $x = 4x^2 - 4x + 1$

or $4x^2 - 5x + 1 = 0$

$(4x-1)(x-1) = 0$

$x = \frac{1}{4}$ or 1

But $\ln(\underbrace{2\cdot\frac{1}{4}-1}_{-\frac{1}{2}})$ not defined

$\rightarrow$ $x = 1$

(22)

$$2\ln(x+2) - \frac{1}{2}\ln x^4 = 1$$

$$\ln(x+2)^2 - \ln(x^4)^{\frac{1}{2}}$$

$$= \ln(x+2)^2 - \ln x^2 = \ln\frac{(x+2)^2}{x^2} = 1$$

$$= \ln e$$

$$\left(\frac{x+2}{x}\right)^2 = e \quad \Rightarrow \quad \frac{x+2}{x} = \pm\sqrt{e}$$

$$\Rightarrow \frac{2}{x} = \pm\sqrt{e} - 1$$

$$\Rightarrow x = \frac{2}{-1\pm\sqrt{e}}$$

Both within the domain of $\ln(x+2)$ & $\ln x^4$

(16) $f(x)=(\ln x)^3$ domain $(0,\infty)$

$f'(x)=3(\ln x)^2\cdot\frac{1}{x}$.

(14) $f(x)=\cos(\ln x)$ domain $= (0,\infty)$ = domain of $\ln x$

since there is no restriction on the domain of cos.

$f'(x)=-\sin(\ln x)\cdot\frac{1}{x}$.

(20) $\int \sec(\frac{1}{2}\pi x)\,dx$

$= \frac{2}{\pi}\int \sec u\,du$ $\quad u=\frac{\pi x}{2} \rightsquigarrow du=\frac{\pi}{2}dx \rightsquigarrow dx=\frac{2}{\pi}du$

$= \frac{2}{\pi}\ln|\sec u+\tan u|+C$

see 7.3.3.

$= \frac{2}{\pi}\ln\left|\sec\left(\frac{\pi x}{2}\right)+\tan\left(\frac{\pi x}{2}\right)\right|+C$ //

(28) $\int \frac{x^2}{2x^3-1}\,dx$

$= \frac{1}{6}\int \frac{\cancel{x^2}}{u}\cdot\frac{1}{\cancel{x^2}}\,du$ $\quad u=2x^3-1$, $du=6x^2dx \rightsquigarrow dx=\frac{1}{6x^2}du$.

$=\frac{1}{6}\ln|u|+C=\frac{1}{6}\ln|2x^3-1|+C$ //

(30) $\int \frac{\sec(2x)\tan(2x)}{1+\sec(2x)}\,dx$ $\quad u=1+\sec(2x)$

$= \int \frac{\cancel{\sec(2x)\tan(2x)}}{u}\cdot\frac{dx}{\cancel{2\sec(2x)\tan(2x)}}$ $\quad du=2\sec(2x)\tan(2x)dx$ or $dx=\frac{du}{2\sec(2x)\tan(2x)}$

$=\ln|u|+C$

$=\ln|1+\sec(2x)|+C$ //