Hw 13

§7-4 : 16, 18, 24, 34, 40, 62, 68

§7-5 : 16, 26, 34, 62

§7-6 : 8, 14, 20.

§7-4.

※16. $y = x^2 \cdot e^x - x \cdot e^{x^2}$

$$\frac{dy}{dx} = 2x \cdot e^x + x^2 \cdot e^x - \left(e^{x^2} + x \cdot e^{x^2} \cdot 2x \right)$$

$$= \underline{2x \cdot e^x + x^2 \cdot e^x - e^{x^2} - 2x^2 \cdot e^{x^2}}$$

※18.

$$y = \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$\frac{dy}{dx} = \frac{(e^{2x}+1) \cdot (e^{2x}-1)' - (e^{2x}-1) \cdot (e^{2x}+1)'}{(e^{2x}+1)^2}$$

$$= \frac{(e^{2x}+1) \cdot e^{2x} \cdot 2 - (e^{2x}-1) \cdot e^{2x} \cdot 2}{(e^{2x}+1)^2}$$

$$= \frac{2 \cdot e^{2x} \cdot [(e^{2x}+1) - (e^{2x}-1)]}{(e^{2x}+1)^2} = \underline{\frac{4 \cdot e^{2x}}{(e^{2x}+1)^2}}$$

※24.

$$f(x) = \ln(\cos e^{2x})$$

$$f'(x) = \frac{(\cos e^{2x})'}{\cos e^{2x}} = \frac{-\sin e^{2x} \cdot (e^{2x})'}{\cos e^{2x}} = -\tan(e^{2x}) \cdot e^{2x} \cdot 2$$

$$= \underline{-2 \cdot e^{2x} \cdot \tan(e^{2x})}$$

※34

$$\int e^{\ln x}\,dx = \int x\,dx = \underline{\frac{1}{2}x^2 + C}$$

※40.

$$\int \frac{\sin(e^{-2x})}{e^{2x}}\,dx = \int \sin u \cdot \underline{e^{-2x}\,dx} = \int \frac{-1}{2} \cdot \sin u\,du$$

$$= \frac{-1}{2} \cdot (-\cos u) + C$$

$$= \frac{1}{2}\cos u + C$$

$$= \underline{\frac{1}{2}\cos e^{-2x} + C}$$

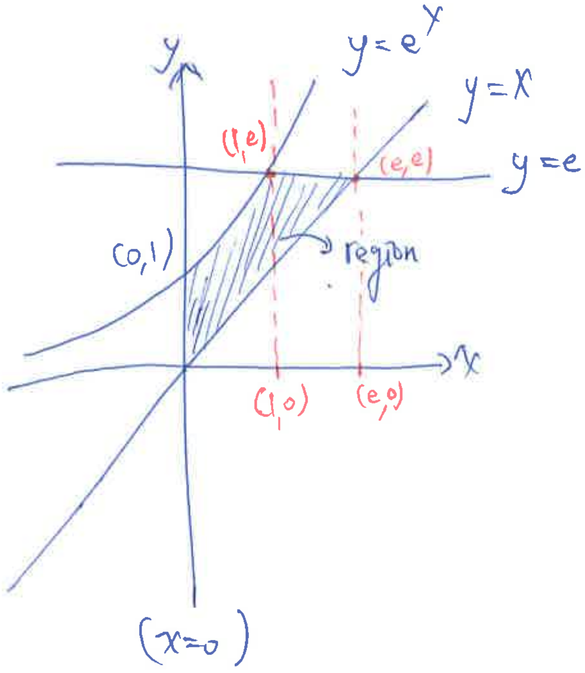
$u = e^{-2x}$

$du = e^{-2x} \cdot (-2)\,dx$

$\frac{-1}{2}\,du = \underline{e^{-2x} \cdot dx}$

※62.

$$y=e^x,\quad y=e,\quad y=x,\quad x=0$$



$$\int_0^1 e^x - x\,dx + \int_1^e e - x\,dx$$

$$= e^x - \frac{1}{2}x^2\Big|_0^1 + \left(ex - \frac{1}{2}x^2\right)\Big|_1^e$$

$$= \left(e-\frac{1}{2}\right) - (1-0) + \left(e^2 - \frac{1}{2}e^2\right) - \left(e - \frac{1}{2}\right)$$

$$= \cancel{e} - \cancel{\frac{1}{2}} - 1 + \frac{1}{2}e^2 - \cancel{e} + \cancel{\frac{1}{2}}$$

$$= \frac{1}{2}e^2 - 1$$

※68.

$$f(x) = (x - x^2)\cdot e^{-x}$$

$$f'(x) = (x^2 - 3x + 1)\cdot e^{-x}$$

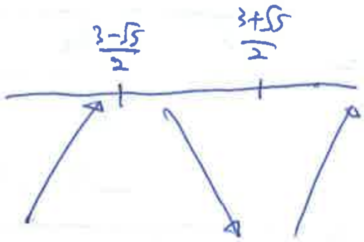
$$f''(x) = -(x^2 - 5x + 4)\cdot e^{-x}$$

① domain: $(-\infty, \infty)$

② Let $f'(x) = 0$

$x^2 - 3x + 1 = 0$

$x = \dfrac{3 \pm \sqrt{5}}{2}$



decrease: $\left(\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right)$

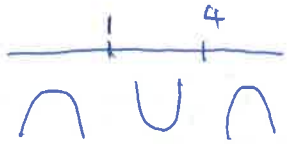
increase: $\left(-\infty, \frac{3-\sqrt{5}}{2}\right)$ or $\left(\frac{3+\sqrt{5}}{2}, \infty\right)$

③ $f\left(\frac{3-\sqrt{5}}{2}\right)$ = local max and absolute max

$f\left(\frac{3+\sqrt{5}}{2}\right)$ = local min.

④ Let $f''(x) = 0$

$x^2 - 5x + 4 = 0$, $x = 1, 4$



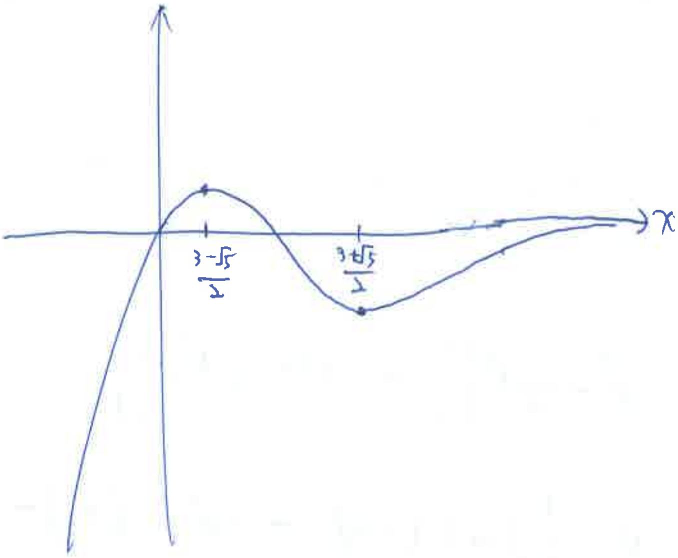
Concave down: $(-\infty, 1)$ or $(4, \infty)$

Concave up: $(1, 4)$

Inflection points: $(1, 0)$

$(4, -12e^{-4})$

\#68. graph:



§7-5

\#16

$$\log_x 2 = \log_3 x \Rightarrow \frac{\ln 2}{\ln x} = \frac{\ln x}{\ln 3} \Rightarrow (\ln x)^2 = \ln 2 \times \ln 3$$

$$\ln x = \pm\sqrt{(\ln 2)\cdot(\ln 3)}$$

$$x = e^{\pm\sqrt{(\ln 2)\cdot(\ln 3)}}$$

\#26.

$$g(x) = \frac{\log_{10} x}{x^2}$$

$$g'(x) = \frac{x^2 \cdot \frac{1}{x} \cdot \frac{1}{\ln 10} - \log_{10} x \cdot 2x}{x^4} = \frac{x \cdot \frac{1}{\ln 10} - \frac{\ln x}{\ln 10} \cdot 2x}{x^4}$$

$$= \frac{1 - 2\ln x}{x^3 \cdot \ln 10}$$

§7-5

※34.

$$\int \frac{\log_5 x}{x}\,dx = \int \frac{1}{\ln 5}\cdot\frac{\ln x}{x}\,dx = \frac{1}{\ln 5}\cdot\int \frac{\ln x}{x}\,dx = \frac{(\ln x)^2}{2\cdot\ln 5} + C$$

$$\int \frac{\ln x}{x}\,dx = \int \ln x\cdot d(\ln x) = \frac{1}{2}(\ln x)^2 + C$$

※62.

$$\int_0^2 p^{x/2}\,dx = \frac{2}{\ln p}\cdot p^{\frac{x}{2}}\Big|_0^2 = \frac{2}{\ln p}\cdot p - \frac{2}{\ln p} = \frac{2p-2}{\ln p}$$

$$\int p^{x/2}\,dx = \int 2\cdot p^u\,du = 2\cdot\frac{1}{\ln p}\cdot p^u + C = \frac{2}{\ln p}\cdot p^{\frac{x}{2}} + C$$

$u = \frac{x}{2}$

$du = \frac{1}{2}\,dx$

§7-6

※8.

$$y = C\cdot e^{kt} \Rightarrow q\cdot C = C\cdot e^{kp}$$

$$\Rightarrow q = e^{kp}$$

$$\Rightarrow kp = \ln q$$

$$p = \frac{1}{k}\ln q$$

※ 14.

$$\frac{ds}{dx} = -\frac{s}{V}$$

Let $s = f(x)$

$f'(x) = \frac{-1}{V} f(x)$, by theorem 7.6.1 , $\cdot\, k = \frac{-1}{V}$

$\Rightarrow \quad f(x) = C \cdot e^{\frac{-1}{V}x}$

we want $f(x) = \frac{1}{2}C \Rightarrow e^{\frac{-1}{V}x} = \frac{1}{2}$

$\Rightarrow \frac{-1}{V}x = \ln\left(\frac{1}{2}\right) = -\ln 2$

$\Rightarrow \frac{x}{V} = \ln 2 \quad \Rightarrow x = V \cdot \ln 2$

$V = 10000$

$\Rightarrow x = 6931$ gallons.

※ 20.

$A(t) = n \cdot e^{kt}$

Let $A(5) = n \cdot e^{5k} = m \Rightarrow e^{5k} = \frac{m}{n} \Rightarrow 5k = \ln\left(\frac{m}{n}\right) \Rightarrow k = \frac{1}{5} \cdot \ln\left(\frac{m}{n}\right)$

$\therefore\ A(t) = n \cdot e^{\frac{1}{5} \cdot \ln\left(\frac{m}{n}\right) \cdot t}$

$A(10) = n \cdot e^{2\ln\left(\frac{m}{n}\right)} = n \cdot e^{\ln\left(\frac{m}{n}\right)^2} = n \cdot \left(\frac{m}{n}\right)^2 = \frac{m^2}{n}$ grams.