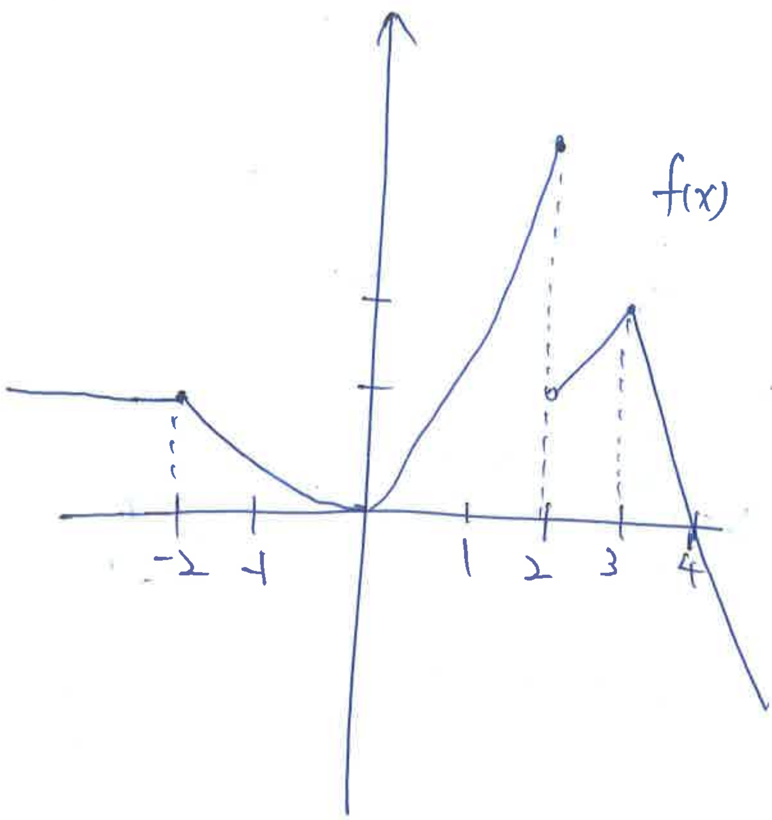
3-1

#22.



(a)

discontinuous:

$x=2$

since

$$\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$$

so,

$x=2$ is not removable.

(jump discontinuous)

(b)

$$\lim_{x\to -2^-} \frac{f(x)-f(-2)}{x-(-2)} = 0 \quad \lim_{x\to -2^+} \frac{f(x)-f(-2)}{x-(-2)} < 0 \Rightarrow f'(-2) \text{ 不存在}$$

$$\lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3} > 0 \quad \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3} < 0 \Rightarrow f'(3) \text{ 不存在}$$

so, $x=-2$ or 3 : continuous

not differentiable

#30.

$$f(x)=\begin{cases} 3x^2 & , x\le 1 \\ 2x^3+1 & , x>1 \end{cases}$$ 求 $f'(1)=$ ____

$f(1)=3$

$$\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1^-}\frac{3x^2-3}{x-1}=\lim_{x\to 1^-}3(x+1)=6$$

$(x<1)$

$$\Rightarrow f'(1)=6$$

$$\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1^+}\frac{2x^3+1-3}{x-1}=\lim_{x\to 1^+}2(x^2+x+1)=6$$

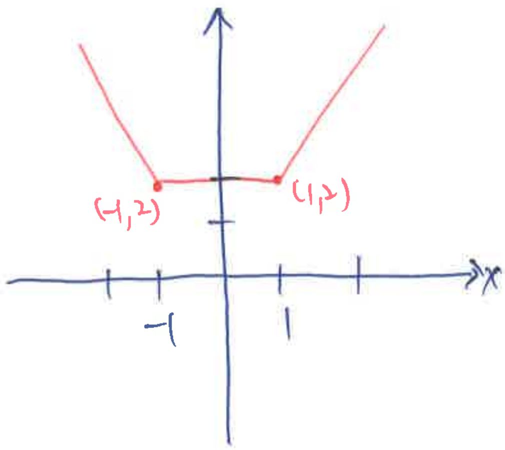
$(x>1)$

#46.

Let $f(x)=|x+1|+|x-1|:\mathbb{R}\to\mathbb{R}$ 或 $f(x)=|x^2-1|$

$f(1)=2$ $f(-1)=2$ $f(0)=2$

So, $f(x)=\begin{cases} 2x & , x\ge 1 \\ 2 & , -1\le x\le 1 \\ -2x & , x\le -1 \end{cases}$



$\Rightarrow f'(1)$ and $f'(-1)$ does not exist,

but $f'(x)$ exists, for all $x\ne\pm 1$,

3-2

※12

$$F(x) = \frac{ax-b}{cx-d}, \quad a, b, c, d : \text{constant.}$$

$$F'(x) = \frac{(ax-b)' \cdot (cx-d) - (ax-b) \cdot (cx-d)'}{(cx-d)^2} = \frac{a \cdot (cx-d) - c \cdot (ax-b)}{(cx-d)^2}$$

$$= \frac{-ad+bc}{(cx-d)^2}$$

※58、

Find the point $(c, f(c))$ where the line tangent to the graph of $f(x) = \frac{x}{x+1}$ is parallel to the secant line that passes through the points $(1, f(1))$ and $(3, f(3))$.

Let $P(1, f(1)) = (1, \frac{1}{2})$ $\quad Q(3, f(3)) = (3, \frac{3}{4})$

the slope of the secant line is $m_{\overleftrightarrow{PQ}} = \frac{\frac{3}{4} - \frac{1}{2}}{3-1} = \frac{\frac{1}{4}}{2} = \frac{1}{8}$

since tangent line is parallel secant line

so, $f'(c) = m_{\overleftrightarrow{PQ}} = \frac{1}{8}$

$$f'(c) = \frac{1}{(c+1)^2} \Rightarrow \frac{1}{(c+1)^2} = \frac{1}{8}$$

$$\Rightarrow (c+1)^2 = 8$$

$$c+1 = \pm 2\sqrt{2}$$

$$c = -1 \pm 2\sqrt{2}$$

① $c = -1 + 2\sqrt{2}$

$\left(-1+2\sqrt{2}, \frac{4-\sqrt{2}}{4}\right)$

② $c = -1 - 2\sqrt{2}$

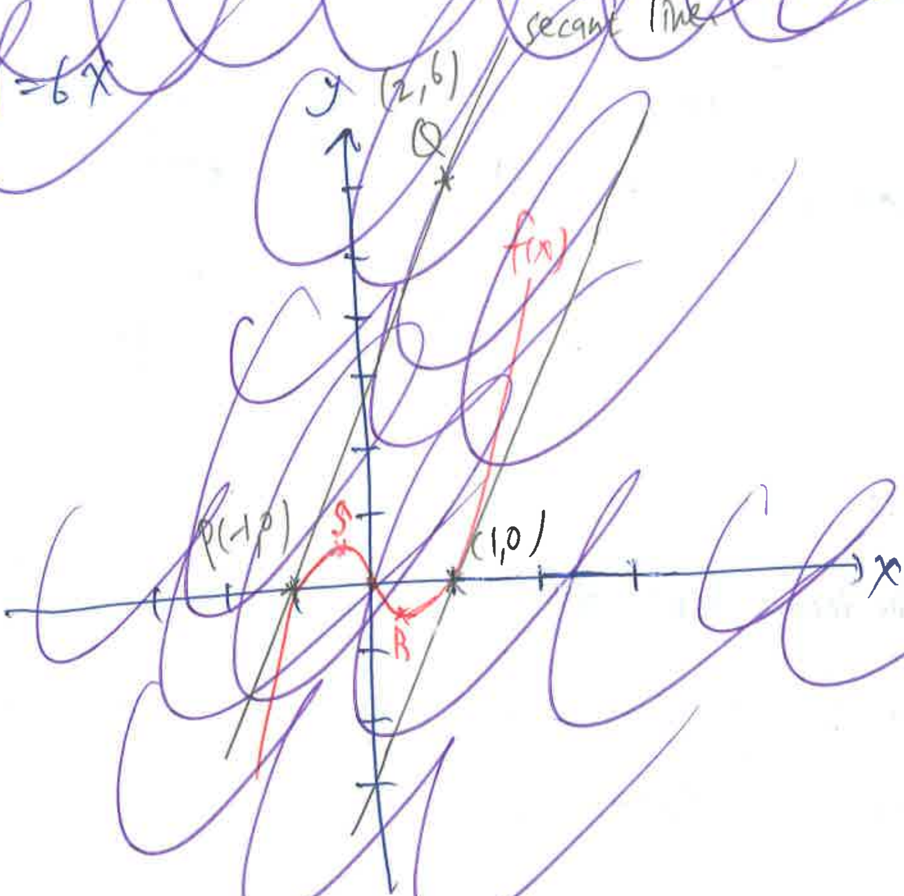
$\left(-1-2\sqrt{2}, \frac{4+\sqrt{2}}{4}\right)$

$f(x) = x^3 - x$

$f'(x) = 3x^2 - 1 = 0 \qquad x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3} \Rightarrow R\left(\frac{\sqrt{3}}{3}, \frac{-2\sqrt{3}}{9}\right)$

$S\left(\frac{-\sqrt{3}}{3}, \frac{2\sqrt{3}}{9}\right)$

$f''(x) = 6x$



3-3

※30.

$f(x) = x^2 - \frac{1}{x^2}$, find $f''(x) =$ ____

$f(x) = x^2 - x^{-2}$

$f'(x) = 2x - (-2)\cdot x^{-3} = 2x + 2\cdot x^{-3}$

$f''(x) = 2 + 2\cdot(-3)\cdot x^{-4} = 2 - 6\cdot x^{-4}$

※50. Find a cubic polynomial P with $p(-1)=0$ $p''(-1)=-2$
$p'(-1)=3$ $p'''(-1)=6$

Let $p(x) = ax^3 + bx^2 + cx + d$

$p'(x) = 3ax^2 + 2bx + c$

$p''(x) = 6ax + 2b$

$p'''(x) = 6a$ 且 $p'''(-1) = 6 \Rightarrow 6a = 6 \Rightarrow a = 1$

$p''(x) = 6x + 2b$ 且 $p''(-1) = -6 + 2b = -2 \Rightarrow b = 2$

$p'(x) = 3x^2 + 4x + c$ 且 $p'(-1) = 3 - 4 + c = 3 \Rightarrow c = 4$

$p(x) = x^3 + 2x^2 + 4x + d$ 且 $p(-1) = -1 + 2 - 4 + d = 0 \Rightarrow d = 3$

so, $p(x) = x^3 + 2x^2 + 4x + 3$