Hw 5

§3-4 ※ 2, 10, 12, 16

§3-5 ※ 10, 16, 20, 24, 26,

§3-6 ※ 6, 18, 22, 32, 50, 70

§3-4

※2. Find the rate of change of the volume of a cube with respect to the length s of a side. What is the rate when s=4.

※10. Find the rate of change of the area A of a circle with respect to

(a) the diameter d (b) the circumference C.

※12. The dimensions of a rectangle are changing in such a way that the area of the rectangle remains constant. Find the rate of change of the height h with respect to the base b.

※16. Find the rate of change of the product $f(x)\cdot g(x)\cdot h(x)$ with respect to x at $x=1$ given that $f(1)=0$, $g(1)=2$, $h(1)=-2$,

$f'(1)=1$, $g'(1)=-1$, $h'(1)=0$.

§3-5

※10. Differentiate the function.

$$f(x)=\left(x^2+\frac{1}{x^2}\right)^3$$

※16.

$$f(x) = \left(\frac{4x+3}{5x-2} \right)^3$$

※20.

$$f(x) = \left[(6x+x^5)^{-1} + x \right]^2$$

※24. Find $\frac{dy}{dx}$, at $x=0$

$y = u^3 - u + 1$, $u = \frac{1-x}{1+x}$, $x = 5t+2$

※26. Find $\frac{dy}{dt}$

$y = 1+u^2$, $u = \frac{1-7x}{1+x^2}$, $x = 5t+2$

§3-6.

※6. Differentiate the function

$y = 3t^2 \cdot \tan t$

※18. Find the second derivative

$y = \sin^5 3t$

※22. Find the second derivate

$$y = \frac{\sin x}{1-\cos x}$$

※32. Find an equation for the line tangent to the curve at the point with x coordinate a.

$y = \tan x$, $a = \frac{\pi}{6}$

※50

Find all x in $[0, 2\pi)$ at which

(a) $f'(x) = 0$ (b) $f'(x) > 0$ (c) $f'(x) < 0$

$f(x) = \sin x - \cos x$

※70.

$$f(x) = \begin{cases} 1 + a\cos x, & x \le \frac{\pi}{3} \\ b + \sin\left(\frac{x}{2}\right), & x > \frac{\pi}{3} \end{cases}$$

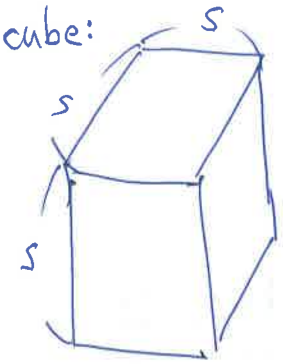
(a) For what values of a and b is f differentiable at $\frac{\pi}{3}$?

(b) Using the values of a and b you found in part (a), sketch the graph of f.

§3-4.

2.

cube:



Volume $= s \times s \times s = s^3$

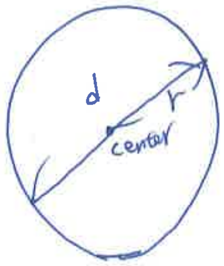
Let $\underline{V = s^3}$

$\frac{dV}{ds} = \underline{3s^2}$

$\left.\frac{dV}{ds}\right|_{s=4} = 3 \times 4^2 = \underline{48}$

10. (a)

circle:



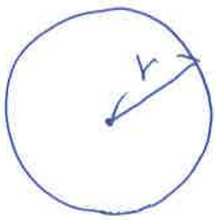
diameter : $d = 2r$, $r = \frac{d}{2}$

area $A = \pi \times \left(\frac{d}{2}\right)^2 = \frac{1}{4}\pi d^2$

Let $\underline{A = \frac{1}{4}\pi d^2}$

$\frac{dA}{dd} = \underline{\frac{1}{2}\pi d}$

(b)



circumference : $C = 2\pi r$, $r = \frac{C}{2\pi}$

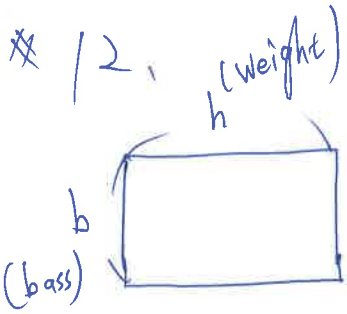
area $A = \pi \times \left(\frac{C}{2\pi}\right)^2 = \pi \times \frac{C^2}{4\pi^2} = \frac{1}{4\pi} \times C^2$

Let $\underline{A = \frac{1}{4\pi} \cdot C^2}$

$\frac{dA}{dC} = \underline{\frac{1}{2\pi} \cdot C}$

§3-4

※ 12.



Area = b×h = constant = C

$$h = \frac{c}{b}$$

Let $h(b) = \frac{c}{b}$

$$\frac{dh}{db} = \frac{0 - c\cdot 1}{b^2} = \frac{-c}{b^2} = \frac{-h\times b}{b^2} = \frac{-h}{b}$$

§3-4

※16.

$F(x) = f(x)\cdot g(x)\cdot h(x)$, $x=1$,

$F'(x) = f'(x)\cdot g(x)\cdot h(x) + f(x)\cdot g'(x)\cdot h(x) + f(x)\cdot g(x)\cdot h'(x)$

$F'(1) = f'(1)\cdot g(1)\cdot h(1) + f(1)\cdot g'(1)\cdot h(1) + f(1)\cdot g(1)\cdot h'(1)$

$= 1\times 2\times(-2) + 0\times(-1)\times(-2) + 0\times 2\times 0$

$= \underline{-4}$

§3-5

10.

$$f(x) = \left(x^2 + \frac{1}{x^2}\right)^3$$

$$f'(x) = 3\cdot\left(x^2+\frac{1}{x^2}\right)^2\cdot\left(x^2+\frac{1}{x^2}\right)'$$

$$= 3\cdot\left(x^2+\frac{1}{x^2}\right)^2\cdot\left(2x+\frac{0-2x}{x^4}\right)$$

$$= 3\cdot\left(x^2+\frac{1}{x^2}\right)^2\cdot\left(2x-\frac{2}{x^3}\right)$$

16.

$$f(x) = \left(\frac{4x+3}{5x-2}\right)^3$$

$$f'(x) = 3\cdot\left(\frac{4x+3}{5x-2}\right)^2\cdot\left(\frac{4x+3}{5x-2}\right)'$$

$$= 3\cdot\left(\frac{4x+3}{5x-2}\right)^2\cdot\frac{4(5x-2)-5(4x+3)}{(5x-2)^2}$$

$$= 3\cdot\frac{(4x+3)^2}{(5x-2)^2}\cdot\frac{-23}{(5x-2)^2} = -69\cdot\frac{(4x+3)^2}{(5x-2)^4}$$

20. $f(x) = \left[(6x+x^5)^{-1}+x\right]^2$

$$= 2\cdot\left[(6x+x^5)^{-1}+x\right]\cdot\left[(6x+x^5)^{-1}+x\right]'$$

$$= 2\cdot\left[(6x+x^5)^{-1}+x\right]\cdot\left[-(6x+x^5)^{-2}\cdot(6+5x^4)+1\right]$$

§3-5

26,

$$y = 1+u^2, \quad u = \frac{1-7x}{1+x^2}, \quad x = 5t+2$$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dx} \cdot \frac{dx}{dt}$$

$$= 2u \cdot \frac{-7(1+x^2) - (1-7x) \cdot 2x}{(1+x^2)^2} \cdot 5 \cdot \quad 2u \cdot \frac{-7-7x^2-2x+14x^2}{(1+x^2)^2} \cdot 5$$

$$= 2 \cdot \frac{1-7x}{1+x^2} \cdot \frac{7x^2-2x-7}{(1+x^2)^2} \cdot 5 = 10 \cdot \frac{(-35t-13) \cdot (175t^2+130t+17)}{[1+(5t+2)^2]^3}$$

§3-6

6.

$$y = 3t^2 \cdot \tan t$$

$$\frac{dy}{dt} = 6t \cdot \tan t + \sec^2 t \cdot 3t^2$$

18.

$$y = \sin^5(3t)$$

$$\frac{dy}{dt} = 5 \cdot \sin^4(3t) \cdot (\sin(3t))'$$

$$= 5 \cdot \sin^4(3t) \cdot \cos(3t) \cdot 3$$

$$= 15 \cdot \sin^4(3t) \cdot \cos(3t)$$

$$\frac{d^2y}{dt^2} = 15(\sin^4(3t))' \cdot \cos(3t) + 15 \cdot (\cos(3t))' \cdot \sin^4(3t)$$

$$= 60 \sin^3(3t) \cdot \cos(3t) \cdot 3 \cdot \cos(3t) + 15 \cdot (-\sin(3t) \cdot 3) \cdot \sin^4(3t)$$

$$= 180 \sin^3(3t) \cos^2(3t) - 45 \cdot \sin^5(3t)$$

22.

$$y = \frac{\sin x}{1-\cos x}$$

$$\frac{dy}{dx} = \frac{(1-\cos x)\cdot\cos x - \sin x\cdot(-(-\sin x))}{(1-\cos x)^2}$$

$$= \frac{\cos x - \cos^2 x - \sin^2 x}{(1-\cos x)^2} = \frac{\cos x - 1}{(1-\cos x)^2} = \frac{-1}{1-\cos x}$$

$$\frac{d^2y}{dx^2} = \frac{0-(-1)\cdot\sin x}{(1-\cos x)^2} = \frac{\sin x}{(1-\cos x)^2}$$

32.

$$y = \tan x, \quad a = \frac{\pi}{6} \Rightarrow \frac{dy}{dx} = \sec^2 x$$

$$m = \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = \sec^2\left(\frac{\pi}{6}\right) = \frac{4}{3}$$

tangent line: $y = \frac{4}{3}\left(x - \frac{\pi}{6}\right) + \tan\frac{\pi}{6}$

$$y = \frac{4}{3}\left(x-\frac{\pi}{6}\right) + \frac{\sqrt{3}}{3}$$

$$y = \frac{4}{3}x - \frac{2\pi}{9} + \frac{\sqrt{3}}{3}$$

§3-6

30.

$f(x) = \sin x - \cos x \quad , \quad 0 < x < 2\pi ,$

$f'(x) = \cos x - (-\sin x) = \sin x + \cos x$

(a) $f'(x) = 0 \qquad \sin x + \cos x = 0$

$\sin x = -\cos x$

$\sin^2 x = \cos^2 x$

$\sin^2 x = 1 - \sin^2 x$

$\sin^2 x = \frac{1}{2} \qquad \sin x = \frac{\sqrt{2}}{2}$ or $\sin x = \frac{-\sqrt{2}}{2}$

$(\because \sin x \neq -\cos x)$

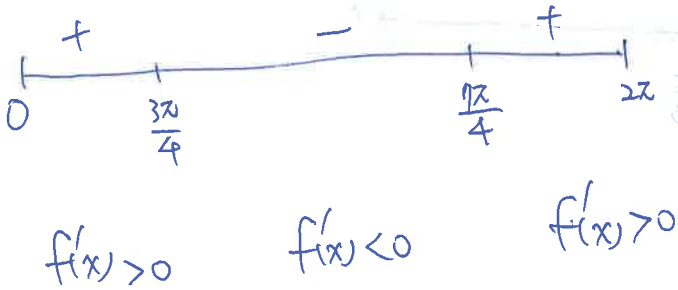
$x = \frac{\pi}{4}, \frac{3\pi}{4}$ or $x = \frac{5\pi}{4}, \frac{7\pi}{4}$

$(\because \sin x \neq -\cos x)$

(不合) (不合)

(b) $f'(x) > 0 \qquad \sin x + \cos x > 0$ on $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

(c) $f'(x) < 0 \qquad \sin x + \cos x < 0$ on $(\frac{3\pi}{4}, \frac{7\pi}{4})$



10.

(a)

$$f(x) = \begin{cases} 1 + a\cos x, & x \le \frac{\pi}{3} \\ b + \sin(\frac{x}{2}), & x > \frac{\pi}{3} \end{cases}$$

since f is differentiable at $x = \frac{\pi}{3}$

$\Rightarrow$ f is continuous at $x = \frac{\pi}{3}$

$$\Rightarrow f'(x) = \begin{cases} -a\sin x, & x \le \frac{\pi}{3} \\ \frac{1}{2}\cos(\frac{x}{2}), & x > \frac{\pi}{3} \end{cases}$$

$$\begin{cases} f(\frac{\pi}{3}) = 1 + a\cdot\cos(\frac{\pi}{3}) = b + \sin(\frac{\pi}{6}) \\ f'(\frac{\pi}{3}) = -a\cdot\sin(\frac{\pi}{3}) = \frac{1}{2}\cos(\frac{\pi}{6}) \end{cases}$$

$$\begin{cases} 1 + \frac{a}{2} = b + \frac{1}{2} & \Rightarrow b = \frac{1}{4} \\ -a\cdot\frac{\sqrt{3}}{2} = \frac{1}{2}\cdot\frac{\sqrt{3}}{2} & \Rightarrow a = \frac{-1}{2} \end{cases}$$

(b)

$$f(x) = \begin{cases} 1 - \frac{1}{2}\cos x, & x \le \frac{\pi}{3} \\ \frac{1}{4} + \sin(\frac{x}{2}), & x > \frac{\pi}{3} \end{cases}$$

graph: