Hw9:

§4-11: 14

§4-12: 16

§5-2: 10, 23

§5-3: 2, 18, 22, 29, 36

§5-4: 4, 8, 24, 32, 46

§4-11:

14.

Estime $f(5.4)$ given that $f(5)=1$ and $f'(x)=\sqrt[3]{2+x^2}$

$$f(x+h) \cong f(x) + f'(x)\cdot h$$

Let $x=5$, $h=0.4$,

$$f(5.4) \cong f(5) + f'(5)\cdot 0.4 = 1 + 3\times 0.4 = 2.2$$

$$f'(5) = \sqrt[3]{2+25} = \sqrt[3]{27} = 3$$

§4-12:

16. $f(x) = x^4 - 7x^2 - 8x - 3$

(a) Show that f has exactly one critical point c in the interval $(2,3)$

(b) Use the Newton-Raphson method to estimate c by calculating x_3. Round off your answer to four decimal places. Does f have a local maximum at c, a local minimum, or neither?

<pf> (a)

$$f'(x) = 4x^3 - 14x - 8$$

$$f''(x) = 12x^2 - 14$$

$\because f'(2) = -4 < 0$, $f'(3) = 58 > 0$
$(\searrow)$ $(\nearrow)$

$\therefore f'(x)$ has a zero in $(2,3)$

and $f''(x) > 0$ on $(2,3)$

$\Rightarrow f'$ has exactly one zero in $(2,3)$ #

$f'(c)=0$, $2<c<3$

(b)

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)} \quad (\because f'(c)=0, \text{ to find } c, \text{ use Newton-Raphson method})$$

$$x_{n+1} = x_n - \frac{4x_n^3-14x_n-8}{12x_n^2-14} = \frac{4x_n^3+4}{6x_n^2-7}$$

$$x_1 = 3 \Rightarrow x_2 = \frac{108+4}{54-7} = \frac{112}{47}$$

$$\Rightarrow x_3 = \frac{4\times\left(\frac{112}{47}\right)^3+4}{6\times\left(\frac{112}{47}\right)^2-7} = \frac{4\times 112^3+4\times 47^3}{6\times 112^2\times 47-47^3\times 7}$$

$$\approx 2.1091$$

$\because f''(x_3) = f''(2.1091) > 0$

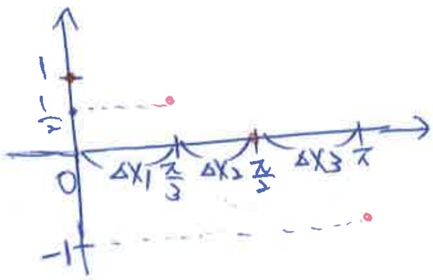
$\therefore f(x)$ has a local minimum at c.

§5-2

10. Calculate $L_f(P)$, $U_f(P)$

$f(x) = \cos x$, $0 \le x \le \pi$, $P = \{0, \frac{\pi}{3}, \frac{\pi}{2}, \pi\}$

$f(0) = \cos 0 = 1$ $\quad f(\frac{\pi}{3}) = \cos\frac{\pi}{3} = \frac{1}{2}$ $\quad f(\frac{\pi}{2}) = \cos\frac{\pi}{2} = 0$, $\quad f(\pi) = \cos\pi = -1$



$$L_f(P) = \frac{1}{2}\times\frac{\pi}{3} + 0\times\frac{\pi}{6} + (-1)\times\frac{\pi}{2} = \frac{\pi}{3}$$

$$U_f(P) = 1\times\frac{\pi}{3} + \frac{1}{2}\times\frac{\pi}{6} + 0\times\frac{\pi}{2} = \frac{5\pi}{12}$$

$\Delta x_1 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$

$\Delta x_2 = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$\Delta x_3 = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

§5-2:

※23) Evaluate $\int_0^1 x^3\,dx$, use $L_f(P)$ and $U_f(P)$.

$x_0=0$, $x_n=1$, $f(x)=x^3$,

Let $P=\{x_0, x_1, x_2, \cdots, x_n\}$,

$$L_f(P)=x_0^3\cdot(x_1-x_0)+x_1^3\cdot(x_2-x_1)+x_2^3(x_3-x_2)+\cdots+x_{n-1}^3\cdot(x_n-x_{n-1})$$

$$U_f(P)=x_1^3\cdot(x_1-x_0)+x_2^3(x_2-x_1)+x_3^3(x_3-x_2)+\cdots+x_n^3(x_n-x_{n-1})$$

$$\because\ x_0<x_1<x_2<\cdots<x_{i-1}<x_i<\cdots<x_n$$

$$\Rightarrow 4x_{i-1}^3\le x_i^3+x_i^2x_{i-1}+x_i\cdot x_{i-1}^2+x_{i-1}^3\le 4x_i^3$$

$$\Rightarrow x_{i-1}^3\le \frac{1}{4}(x_i^3+x_i^2x_{i-1}+x_ix_{i-1}^2+x_{i-1}^3)\le x_i^3,\quad i=1,2,\cdots,n$$

$$\Rightarrow x_{i-1}^3(x_i-x_{i-1})\le \frac{1}{4}\underline{(x_i-x_{i-1})\cdot(x_i^3+x_i^2x_{i-1}+x_ix_{i-1}^2+x_{i-1}^3)}\le x_i^3\cdot(x_i-x_{i-1}),\quad i=1,2,3,\cdots,n$$

$$\Rightarrow x_{i-1}^3(x_i-x_{i-1})\le \frac{1}{4}\cdot(x_i^4-x_{i-1}^4)\le x_i^3\cdot(x_i-x_{i-1}),\quad i=1,2,\cdots,n$$

$$\Rightarrow \sum_{i=1}^n x_{i-1}^3(x_i-x_{i-1})\le \sum_{i=1}^n\frac{1}{4}(x_i^4-x_{i-1}^4)\le\sum_{i=1}^n x_i^3\cdot(x_i-x_{i-1})$$

$$\Rightarrow L_f(P)\le\frac{1}{4}(x_n^4-x_0^4)\le U_f(P),\quad x_n=1,\ x_0=0\ 代入,$$

$$\Rightarrow L_f(P)\le\frac{1}{4}\le U_f(P)$$

by definition 5,2,3

$$\Rightarrow \int_0^1 x^3\,dx=\frac{1}{4}$$

§5-3

※ 2.

$$\int_1^4 f(x)\,dx = 5 \qquad \int_3^4 f(x)\,dx = 7 \qquad \int_1^8 f(x)\,dx = 11$$

(a)

$$\int_4^8 f(x)\,dx = \int_1^8 f(x)\,dx - \int_1^4 f(x)\,dx = 11 - 5 = \underline{\underline{6}}$$

(b)

$$\int_4^3 f(x)\,dx = -\int_3^4 f(x)\,dx = \underline{\underline{-7}}$$

(c)

$$\int_1^3 f(x)\,dx = \int_1^4 f(x)\,dx - \int_3^4 f(x)\,dx = 5 - 7 = \underline{\underline{-2}}$$

(d)

$$\int_3^8 f(x)\,dx = \int_1^8 f(x)\,dx - \int_1^3 f(x)\,dx = 11 - (-2) = \underline{\underline{13}}$$

(e)

$$\int_8^4 f(x)\,dx = -\int_4^8 f(x)\,dx = -\left[\int_1^8 f(x)\,dx - \int_1^4 f(x)\,dx\right] = -(11-5) = \underline{\underline{-6}}$$

(f)

$$\int_4^4 f(x)\,dx = \underline{\underline{0}}$$

§5-3

*18.
$$F(x)=\int_0^x \frac{t-4}{1+t^2}\,dt$$

$$\Rightarrow F'(x)=\frac{x-4}{1+x^2}$$

Let $F'(x)=0 \Rightarrow x-4=0\ ,\ 1+x^2\neq 0$

$\Rightarrow x=4$ (critical point)

$$\Rightarrow F''(x)=\frac{1+x^2-(x-4)\cdot 2x}{(1+x^2)^2}=\frac{-x^2+8x+1}{(1+x^2)^2}$$

$F''(4)=\frac{17}{17^2}=\frac{1}{17}>0 \Rightarrow$ F(x) has a local minimum at $x=4$

*22.

g : differentiable and $g'(x)=\begin{cases} <0 & , x<1 \\ 0 & , x=1 \\ >0 & , x>1 \end{cases}$ and $g(1)=0$

$$G(x)=\int_0^x g(t)\,dt$$

(a) $\because G'(x)=g(x)$: differentiable $\Rightarrow$ G(x) is continuous.

(b) $\because G''(x)=g'(x)$: exist $\Rightarrow$ G(x) is twice differentiable.

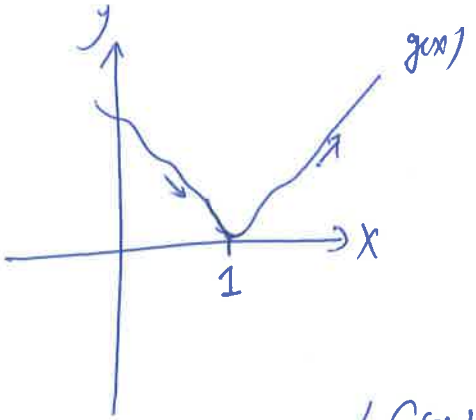
(c) $G'(1)=g(1)=0 \Rightarrow$ G(x) has a critical point at $x=1$

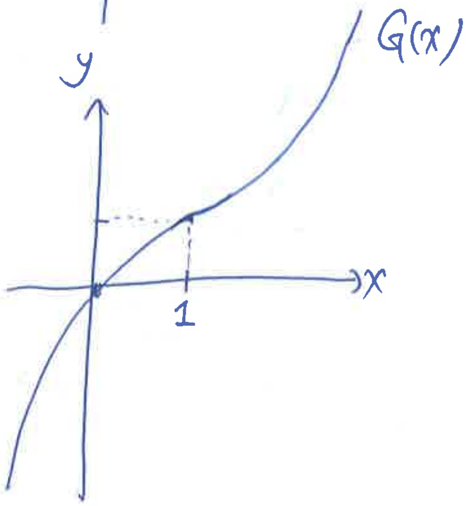
(d) $G''(x)=g'(x)=\begin{cases} <0 & , x<1 \Rightarrow \text{G(x) is concave down for } x<1 \\ >0 & , x>1 \Rightarrow \text{G(x) is concave up for } x>1 \end{cases}$

(e)

$\because G'(x) = g(x) > 0$ for all $x \neq 1$.

$\therefore G(x)$ is increasing function.





※29,

$$F(x) = 2x + \int_0^x \frac{\sin 2t}{1+t^2}\,dt$$

(a) $$F(0) = 2\times 0 + \int_0^0 \frac{\sin 2t}{1+t^2}\,dt = 0$$

(b)

$$F'(x) = 2 + \frac{\sin 2x}{1+x^2} \Rightarrow F'(0) = 2 + \frac{\sin 0}{1+0^2} = 2$$

(c) $$F''(x) = \frac{(1+x^2)\cdot 2\cos 2x - \sin 2x \cdot 2x}{(1+x^2)^2} \Rightarrow F''(0) = \frac{1\times 2}{1} = 2$$

§5-3

※36. f : continuous

$$F(x) = \int_0^x \left[t \int_1^t f(u)\, du \right] dt$$

(a)

$$F'(x) = x \cdot \int_1^x f(u)\, du$$

(b) $F'(1) = 1 \cdot \underbrace{\int_1^1 f(u)\, du}_{=0} = 0$

(c)

$$F''(x) = \int_1^x f(u)\, du + x \cdot \left(\int_1^x f(u)\, du \right)' = \int_1^x f(u)\, du + x \cdot f(x)$$

(d)

$$F''(1) = \underbrace{\int_1^1 f(u)\, du}_{=0} + 1 \cdot f(1) = f(1)$$

§5-4

※4. $\int_1^2 (2x + x^2)\, dx = x^2 + \frac{1}{3}x^3 \Big|_1^2 = \frac{16}{3}$

※8. $\int_1^2 \left(\frac{3}{x^3} + 5x\right) dx = \frac{-3}{2}x^{-2} + \frac{5}{2}x^2 \Big|_1^2 = \frac{69}{8}$

#24.

$$\int_0^{\pi} 3\sin x\, dx = -3\cos x \Big|_0^{\pi} = 3-(-3) = 6$$

#32.

$$\int_{\pi/4}^{\pi/2} \csc x(\cot x - 3\csc x)\, dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc x \cdot \cot x - 3\csc^2 x\, dx = -\csc x + 3\cot x \Big|_{\pi/4}^{\pi/2} = \sqrt{2} - 4$$

#46.

(a)

$$\int_{-4}^{2} (2x+3)\, dx = x^2+3x \Big|_{-4}^{2} = 10-4 = 6$$

(b) $$\int_{-4}^{2} |2x+3|\, dx = \int_{-4}^{\frac{-3}{2}} -(2x+3)\, dx + \int_{\frac{-3}{2}}^{2} 2x+3\, dx$$

$$= (-x^2+3x)\Big|_{-4}^{\frac{-3}{2}} + (x^2+3x)\Big|_{\frac{-3}{2}}^{2}$$

∵ $2x+3=0$

$x = \frac{-3}{2}$

$$= \frac{39}{2}$$