Let $L = q + \mathbb{R}p$ and $L' = q' + \mathbb{R}p'$. Suppose $x, x' \in L \cap L'$ and $x \neq x'$. Therefore, we have

$$x = q + \alpha p \tag{0.1}$$

$$x = q' + \beta p' \tag{0.2}$$

$$x' = q + \alpha' p \tag{0.3}$$

and

$$x' = q' + \beta' p'. \tag{0.4}$$

Since $x \neq x'$, it is necessary that $\alpha \neq \alpha'$ and $\beta \neq \beta'$. (0.1)-(0.1) gives

$$q - q' = \beta p' - \alpha p \tag{0.5}$$

and (0.3)-(0.4) gives

$$q - q' = \beta' p' - \alpha' p \tag{0.6}$$

These two equations implies that $(\beta - \beta')p' = (\alpha - \alpha')p$. Since $\alpha \neq \alpha'$ and $\beta \neq \beta'$, it implies that $p = \lambda p'$, where $\lambda = \frac{\beta - \beta'}{\alpha - \alpha'}$ and therefore $\mathbb{R}p = \mathbb{R}p'$. Then either (0.5) or (0.6) implies that $q - q' \in \mathbb{R}p$.