Pre-Calculus Review

· Elementary Notations

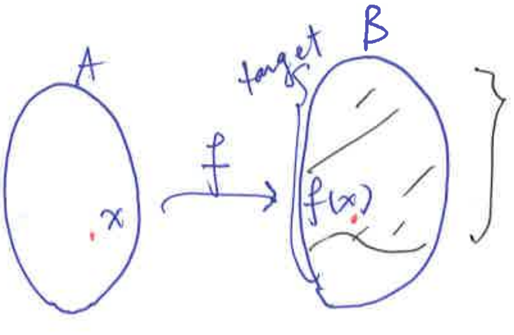
$\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, (a,b), $[a,b)$, $[a,b]$, ...

Set := a collection of elements (!!)

③ Functions

A function is a rule, between two "sets" of elements, that assigns to each element of a set A (domain) unambiguisly to an element of another set B (target). $x, y, \ldots$; $f(x), f(y) \ldots$

$$f : A \longrightarrow B$$



Range of $f = \{ f(x) \mid x \text{ in } A \}$
$R(f)$

e.g. 1. $f : \mathbb{N} \to \mathbb{N}$
$f(n) = n+1$

2. $f : \mathbb{R} \to \mathbb{R}$
$f(x) = x^2$
$R(f) = \mathbb{R}^+$

3. $f : \mathbb{R} \to \mathbb{R}$
$f(x) = \sin(\cos x)$
$R(f) = [-1, 1]$

②

*Piecewise Functions: functions that are separately defined on each subdivision of the domain.

eg.
1. $f: \mathbb{R} \to \mathbb{R}$
$f(x) = \begin{cases} x^2 ; & x \geq 0 \\ x, & x < 0 \end{cases}$

1. $f: \mathbb{R} \to \mathbb{R}$

$f(x) = \begin{cases} 2x ; & x \geq 0 \\ 1 - x ; & x < 0 \end{cases}$

2. $f: \mathbb{R} \to \mathbb{R}$

$f(x) = \begin{cases} x ; & x \geq 0 \\ -x ; & x < 0 \end{cases}$

(actually $f(x) = |x|$)

3. f:

Rmk: In this class, we're mainly interested in $f: A \to B$ where A, B are subsets from $\mathbb{R}$, called "single variable real valued function"

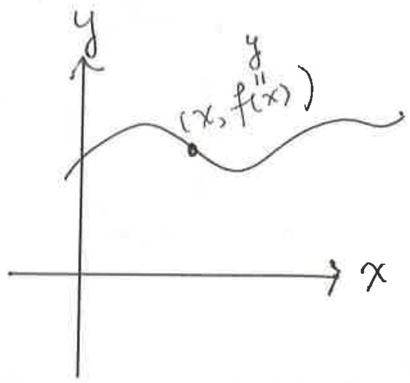
$f(x)$ is called the "value of f at x"

* Graph of $f: \mathbb{R} \to \mathbb{R}$
↳ set of pairs $(x, f(x))$ on $\mathbb{R} \times \mathbb{R}$ $(\mathbb{R}^2)$
plot of

1. $f(x) = 4$ 2. $f(x) = x$ 3. $f(x) = x^2$

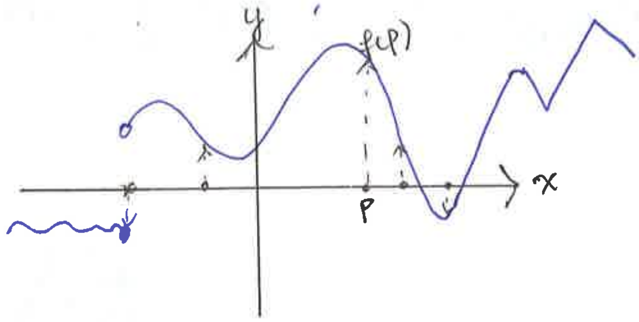
4. $f(x) = \begin{cases} x^2 ; & x \geq 0 \\ x ; & x < 0 \end{cases}$

5. $f(x) = \begin{cases} 2x ; & x \geq 0 \\ 1 - x ; & x < 0 \end{cases}$



Recall: graphs of trigonometric functions
exponential / log "

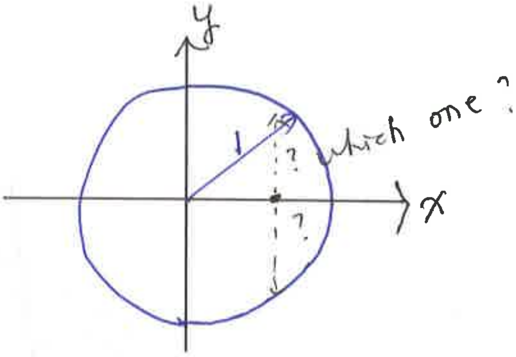
Graph is the geometric interpretation of a function



at each point p on x axis, we can find a unique value f(p) by "looking up or down"

Q Is there any function whose graph has points (3, 4) and (3, 7) ?

Not every "curve" arises from graph of a function



unit circle: all points (x, y) such that $x^2+y^2=1$,

can't be re-written as $y =$ a single formula involving x only

* Some conventions

. $f(x) = \sqrt{x}$

$x \geq 0$ required

and impose $f(x) \geq 0$ (ie $\sqrt{4} = 2$, not ± 2)

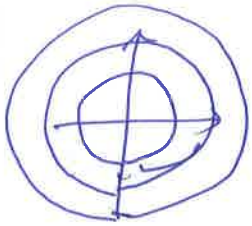
* Curves in $\mathbb{R}^2$

"traces of $P(x,y) - c = 0$"

Ⓐ Given by an equation $P(x,y) = c$, c is a constant of x & y

(all points (x,y) satisfying $P(x,y)=c$)

① $P(x,y) = x^2 + y^2$



$P(x,y) = 1 \rightarrow$ circle of radius 1

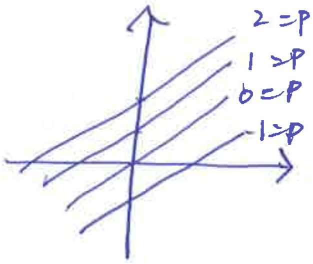
$\quad 2 \rightarrow$ " " 2

$\quad 3 \rightarrow$ " " 3

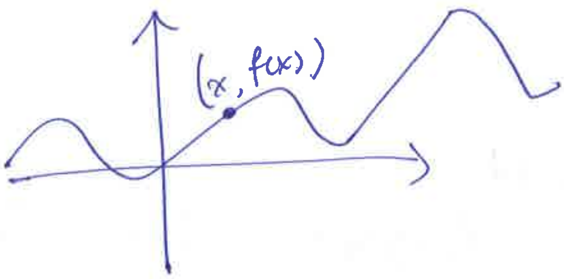
⋮

② $P(x,y) = y - x$

$P(x,y) = -1, 0, 1, 2, \ldots$



Ⓑ Given by graph of a function $y = f(x)$



curves of type Ⓑ are always curves of type Ⓐ, namely $P(x,y) = y - f(x) = 0$, not conversely

(A is B A is not)

* Elementary Functions used in this class

⑤

- polynomials 多項式 ($f: \mathbb{R} \to \mathbb{R}$)

$$f(x) = a_0 + a_1 x + \cdots + a_n x^n \qquad (a_1 = \cdots = a_n = 0 \Rightarrow \text{constant})$$

- Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad \text{where } p(x), q(x) \text{ are polynomials}$$

Domain : $\mathbb{R} \setminus \{q(x) = 0\}$

- Root. $f(x) = \sqrt[n]{x}$

- Exponentials 指數 & Logarithmic 對數

$$f(x) = a^x, \quad \overset{\text{base}}{a} \geq 0$$

particularly important $a = \text{"}e\text{"} \approx 2.71\cdots$

skip for insufficient time

Rmk // what is a^x when x not $\overset{\text{positive}}{\text{integer}}$?

x positive integer $\to$ a times itself x times

x negative " $\to$ $\dfrac{1}{a \text{ times } -x \text{ times}}$

$x = \dfrac{a^k}{b^k}$ integers $\in \mathbb{Q}$ $\to$ $\sqrt[b]{x^a}$

x any real # $\to$ take x_n (in $\mathbb{Q}$) and x_n "approaches" x as n "approaches" ∞

$\therefore$ a^x = "limit" of a^{x_n}

$f(x) = \log_a x$ (ie $a^{f(x)} = x$)

$a = e \rightarrow f(x) = \ln x$ $(x > 0)$

- Factorials $f : \mathbb{N} \rightarrow \mathbb{N}$

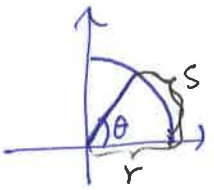
$$f(n) = n! := n \cdot (n-1) \cdot (n-2) \cdots 1$$

(there is a generalized "factorial function" defined on $\mathbb{R}$)

- Trigonometric

$$f(x) = \begin{cases} \left.\begin{array}{l} \sin x \\ \cos x \end{array}\right\} \mathbb{R} \rightarrow [-1, 1] \\ \tan x = \sin x / \cos x \; : \mathbb{R} \setminus \{ \frac{\text{odd.\#} \cdot \pi}{2} \} \rightarrow \mathbb{R} \\ \sec x = 1/\cos x \; : \mathbb{R} \setminus \{ \quad " \quad \} \rightarrow \mathbb{R} \\ \csc x = 1/\sin x \; : \mathbb{R} \setminus \{ n\pi \} \rightarrow \mathbb{R} \end{cases}$$

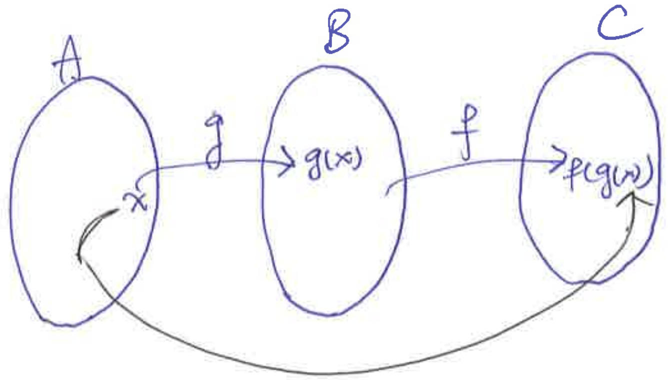
unit of angle : radian



$\theta = \frac{s}{r}$

* Review basic trig. identities.

* Compositions of Functions



$$f \circ g\,(x) := f(g(x))$$

" composition of f with g

eg.

1. $g(x) = 1 - x$ $\qquad f \circ g\,(x) = (1-x)^2 = 1 - 2x + x^2 \; : \mathbb{R} \to \mathbb{R}$

   $f(x) = x^2$

Be careful w/ domain & range!!

2. $g(x) = x - 2$ $\qquad f \circ g(x) = \dfrac{1}{x+1} \; : \mathbb{R} \setminus \{-1\} \to \mathbb{R}$

   $f(x) = \dfrac{1}{x+3}$

3. $g(x) = \sin x$ $\qquad$ domain of $f \circ g$?

   $f(x) = \sqrt{x}$

4. $g(x) = \sin x$ $\qquad f \circ g$ undefined !

   $f(x) = \sqrt{x-2}$