IV Traces vs Graphs

Compare B1. B2 or C1 with A1 ~ A4. fundamental difference?

Geometrically, in type B & C. at any point $(x, y, 0)$ in the domain, we can "unambiguisouly" tell the z value by looking "up or down". (Not so in type A)

Algebraically, equations in type B & C can all be rewritten unambiguisouly into $z = \dots$ (in type A there will be $\pm$ involved)

In other words, z-plots in type B & C are determined by first two components of the coordinates, they're "graphs" of a functions.

Precise Definitions:

Recall,

Def1: The trace (or plot) of an equation $P : \mathbb{R}^n \to \mathbb{R}$

$= \{ \vec{x} \in \mathbb{R}^n \mid P(\vec{x}) = 0 \}$

eg IV1 Sphere in $\mathbb{R}^3$ of radius 2 is the trace of

$P(x, y, z) = x^2 + y^2 + z^2 - 4$

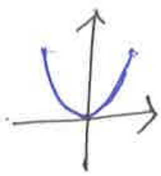
(also called "level set of P at 0)

Def'n a function $f: A \to B$ is a rule that associates to each element $x \in A$ unambiguously an element $f(x) \in B$

Def'n For $f: \mathbb{R}^{n-1} \to \mathbb{R}$, the graph of f is defined to be
$$Gr(f) = \{ (x_1, \ldots, x_{n-1}, f(x_1, \ldots, x_{n-1})) \mid (x_1, \ldots, x_{n-1}) \in \mathbb{R}^{n-1} \}$$

eg VI 2 $\quad f: \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2$

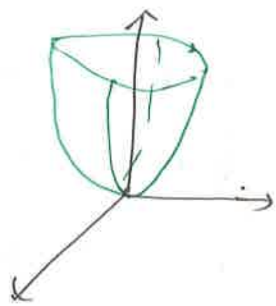
$$Gr(f) = \{ (x, x^2) \mid x \in \mathbb{R} \} \subset \mathbb{R}^2$$



VI 3 $\quad f: \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = x^2 + y^2$

(elliptical paraboloid)

$$Gr(f) = \{ (x, y, x^2+y^2) \mid (x, y) \in \mathbb{R}^2 \} \subset \mathbb{R}^3$$



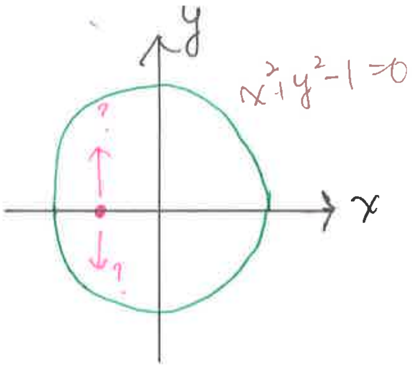
A graph is always a trace:

for $f: \mathbb{R}^{n-1} \to \mathbb{R}$,
$Gr(f)$ = Trace defined by $P(x_1, \ldots, x_n) = x_n - f(x_1, \ldots, x_{n-1})$

In VI 3, $Gr(f)$ = Trace of $P(x, y, z) = z - f(x, y)$
$= z - (x^2 + y^2)$

$(\because z = x^2 + y^2)$

Converse is NOT true:



Not graph of any function.

When are traces graphes ?
(if no obvious sketch available)

ANS: Implicit Function Theorem.

10/22 ends