

Homework I. Solutions

§1.5.

$$*12. f(x) = \frac{1}{x-2}$$

$$f(a+h) = \frac{1}{a+h-2}, \quad f(a) = \frac{1}{a-2}$$

$$\frac{f(a+h) - f(a)}{h} = \frac{1}{h} \left[\frac{1}{a+h-2} - \frac{1}{a-2} \right] = \frac{1}{h} \frac{(a-2) - (a+h-2)}{(a+h-2)(a-2)} = \frac{-h}{h(a+h-2)(a-2)} = \frac{-1}{(a+h-2)(a-2)}$$

$$*18. f(x) = \frac{x}{|x|}$$

for all $x > 0$, $f(x) = 1$

$$*22. g(x) = \sqrt{x} + 5$$

$$\text{dom}(g) = [0, \infty)$$

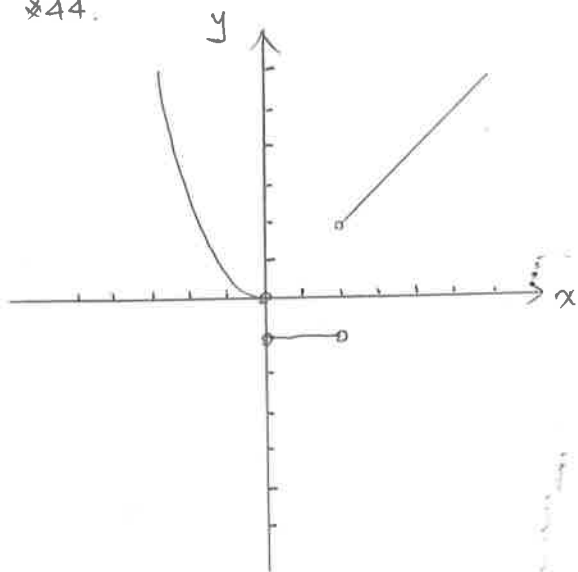
$$\text{range}(g) = [5, \infty)$$

$$*30. g(x) = \frac{1}{\sqrt{4-x^2}}$$

$$\text{dom}(g) = (-2, 2)$$

$$\text{range}(g) = \left[\frac{1}{2}, \infty \right)$$

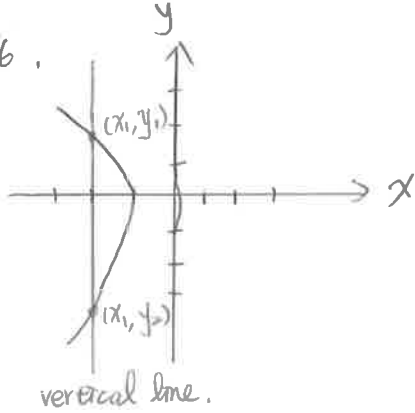
*44.



$$\text{domain} : (-\infty, 0) \cup (0, 2) \cup (2, \infty)$$

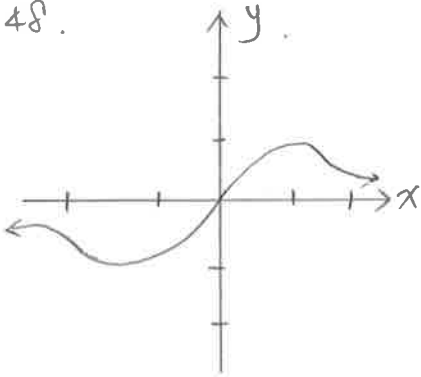
$$\text{range} : \{-1\} \cup (0, \infty)$$

*46.



(p. 26) : A curve C which intersects each vertical line at most once is the graph of a function.
 \Rightarrow the curve is NOT the graph of a function

*48.



It is the graph of a function.
 domain: $(-\infty, \infty)$
 range: $(-1, 1)$

§1.7.

$$*6. \left(\frac{f+g}{f}\right)(-1) = \frac{2+3+1+2(1-1)}{2+3+1} = \frac{6}{6} = 1.$$

*12.

$$\textcircled{1} (f+g)(x) = \sin^2 x + \cos^2 x = \sin^2 x + (1 - \sin^2 x) = 1 - \sin^2 x = \cos^2 x$$

$$\text{dom}(f+g) = (-\infty, \infty)$$

$$\textcircled{2} (f-g)(x) = \sin^2 x - \cos^2 x = 3\sin^2 x - 1$$

$$\text{dom}(f-g) = (-\infty, \infty)$$

$$\textcircled{3} (f \cdot g)(x) = \sin^2 x \cos^2 x$$

$$\text{dom}(f \cdot g) = (-\infty, \infty)$$

$$\textcircled{4} \left(\frac{f}{g}\right)(x) = \frac{\sin^2 x}{\cos^2 x}$$

$$\text{dom}\left(\frac{f}{g}\right) = \left\{ x : x \neq \frac{2n+1}{4}\pi, n = 0, \pm 1, \pm 2, \dots \right\}$$

$$\cos 2\left(\frac{2n+1}{4}\pi\right) = 0, n = 0, \pm 1, \pm 2, \dots$$

$$\ast 28. (f \circ g)(x) = f(g(x))$$

$$= \frac{1}{x^2 - 1}$$

$$\text{dom}(f \circ g) = \{x \neq \pm 1\}$$

$\ast 30.$

$$(f \circ g)(x) = \sqrt{1 - 2 \cos x}$$

$$\text{dom}(f \circ g) = \left[\frac{\pi}{3}, \frac{5}{3}\pi \right]$$

$\ast 34.$

$$(f \circ g \circ h)(x) = f(g(h(x)))$$

$$= f\left(\frac{1}{x^2 + 1}\right) = \frac{\frac{1}{x^2 + 1} + 1}{\frac{1}{x^2 + 1}} = \frac{(x^2 + 1)(x^2 + 2)}{x^2 + 1} = x^2 + 2$$

$$\text{dom}(f \circ g \circ h) = (-\infty, \infty)$$