HW#2. Solutions

§2.1

*1.

(a) $\lim_{x\to 2^-} f(x) = 2$.

(b) $\lim_{x\to 2^+} f(x) = -1$

(c). $\lim_{x\to 2} f(x)$ DNE

(d) $f(2) = -3$.

*2.

(a) $\lim_{x\to 3^-} f(x) = -4$

(b) $\lim_{x\to 3^+} f(x) = -4$

(c) $\lim_{x\to 3} f(x) = -4$

(d) $f(3) = 2$

*6.

(a) $\lim_{x\to 1^-} f(x) = 1$

(b) $\lim_{x\to 1^+} f(x) = -2$

(c) $\lim_{x\to 1} f(x)$ DNE

(d) $f(1) = -2$

*12.

when $c = 3$. $\lim_{x\to c} f(x)$ DNE.

*24.

$$\lim_{x\to 3} \frac{x^2-6x+9}{x-3} = \lim_{x\to 3} \frac{(x-3)^{\cancel{2}}}{\cancel{x-3}} = \lim_{x\to 3} x-3 = 0$$

*36.

$$\lim_{x\to 1} \frac{x^2+1}{x^2-1} \quad \text{DNE}$$

*42.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x) = 2$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^2+1) = 2$$

$$\Rightarrow \lim_{x\to 1} f(x) = 2$$

~~HW#2 Solutions~~

§ 2.3

(20) $$\lim_{x\to -2} \frac{(x^2-x-6)^2}{x+2} = \lim_{x\to -2} \frac{(x+2)^{\cancel{2}}(x-3)^2}{\cancel{x+2}} = 0$$

(25) $$\lim_{h\to 0} \frac{1-1/h^2}{1-1/h} = \lim_{h\to 0} \frac{h^2-1}{h^2-h} = \lim_{h\to 0} \frac{(h+1)\cancel{(h-1)}}{h\cancel{(h-1)}} \quad \text{DNE}$$

$$\text{"="} \ \frac{1}{0}$$

(26) $$\lim_{h\to 0} \frac{1-1/h^2}{1+1/h^2} = \lim_{h\to 0} \frac{h^2-1}{h^2+1} = -1$$

(27) $$\lim_{h\to 0} \frac{1-1/h}{1+1/h} = \lim_{h\to 0} \frac{h-1}{h+1} = -1$$

(28) $$\lim_{h\to 0} \frac{1+1/h}{1+1/h^2} = \lim_{h\to 0} \frac{h^2+h}{h^2+1} = 0$$

(38) $$\lim_{x\to -4} \left(\frac{2x}{x+4} - \frac{8}{x+4}\right) = \lim_{x\to -4} \left(\frac{2x-8}{x+4}\right)$$

$$= \lim_{x\to -4} \frac{2(x-4)}{x+4} \ \text{"="} \ \frac{-16}{0} \quad \text{DNE}$$

(42) $f(x) = x^3$

ⓐ $$\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3} \frac{x^3-3^3}{x-3} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{x-3}}$$

$$= 27$$

ⓑ $\displaystyle\lim_{x\to 3} \frac{f(x)-f(2)}{x-3}$ DNE since $f(3)-f(2) \neq 0$.

ⓒ $$\lim_{x\to 3} \frac{f(x)-f(3)}{x-2} = \frac{f(3)-f(3)}{1} = 0$$

ⓓ $$\lim_{x\to 1} \frac{\overbrace{f(x)-f(1)}^{=x^3-1^3}}{x-1}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}} = 3$$

(52) (a) if $f(x) \geq g(x)$, $|f(x)-g(x)| = f(x)-g(x)$

and $\frac{1}{2}\{[f(x)+g(x)]+|f(x)-g(x)|\} = f(x)$

if $f(x) < g(x)$, $|f(x)-g(x)| = g(x)-f(x)$

and " " $= g(x)$

which are precisely definition of $\max\{f(x), g(x)\}$

i.e. $\max\{f(x), g(x)\} = \begin{cases} f(x) : f(x) \geq g(x) \\ g(x) : f(x) < g(x) \end{cases}$

(b) $\min\{f(x), g(x)\} = \begin{cases} f(x) : f(x) < g(x) \\ g(x) : f(x) \geq g(x) \end{cases}$

$\therefore$ $\frac{1}{2}\{[f(x)+g(x)]-|f(x)-g(x)|\}$ will do.

(check it!)

(53)

$\lim h(x) = \lim_{x\to c} \frac{1}{2}\{[f(x)+g(x)] - |f(x)-g(x)|\}$

$|\;|$ is continuous and commutes w/ limit $\rightarrow$ $= \frac{1}{2}\{\lim_{x\to c} f(x) + \lim_{x\to c} g(x) - |\underbrace{\lim_{x\to c} f(x) - \lim_{x\to c} g(x)}_{\to 0}|\}$

$= \frac{1}{2}(L+L) = L$

Similarly $\lim_{x\to c} H(x) = \lim_{x\to c} \frac{1}{2}\{f(x)+g(x)+|f(x)-g(x)|\}$

$= \frac{1}{2}\{L+L+|\lim_{x\to c} f(x) - \lim_{x\to c} g(x)|\} = L$ //