Names and Student IDs: _____

Homework 1 Calculus 1

1. Given an ordered set $(S, <)$, $\emptyset \neq E \subset S$, and $y \in S$ an upper (lower) bound of E. Prove that if $y \in E$, then $y = \sup$ (inf) E. The element y is called the maximal (minimal) element of E.

2. Prove, that for an ordered set $(S, <)$, if S has GLB (Greatest Lower Bound Property), then S has LUB (Least Upper Bound Property). We therefore conclude that LUB and GLB are two equivalent properties.

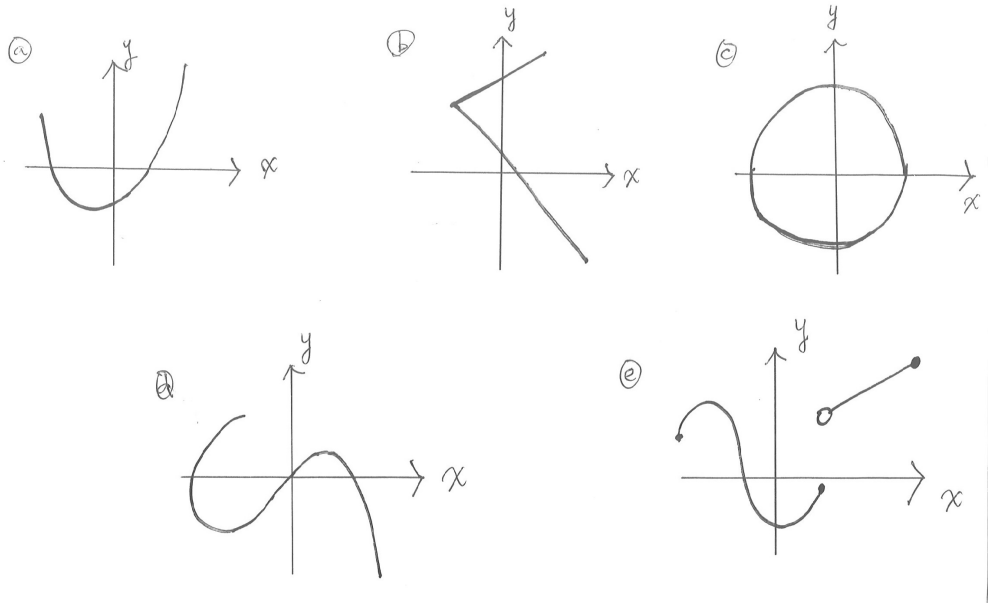
3. Rudin, Exercise 1, p21 and Rudin, Exercise 2, p22.

4. Rudin, Exercise 5, p22.

5. Prove, using $\epsilon - \delta$ definition, that
   (a)
   $$\lim_{x \to 1} \frac{x}{x+1} = \frac{1}{2}.$$
   (b)
   $$\lim_{x \to \infty} \frac{2x^2}{4x^2 + 3x - 1} = \frac{1}{2}.$$

6. Prove that if $f(x) \to L$ and $g(x) \to M \neq 0$ as $x \to c$, then
   $$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}.$$
   (Hint: first prove that $\lim_{x \to c} \frac{1}{g(x)} = \frac{1}{M}$.)

7. Determine, with sufficient reasons, whether each of the following curves is a graph of some function.



8. Salas 2.3: 18, 21, 26, 30, 43, 44, 47, 49, 50.

9. Salas 2.5: 5, 8, 13, 28.