Names and Student IDs: _____

Homework 10 Calculus 1

1. Recall that a function is *convex* if for all $a < b$, we have

$$f(\lambda a + (1-\lambda)b) \leq \lambda f(a) + (1-\lambda)f(b) \ \ \forall \lambda \in [0,1].$$

(The inequality says that over any interval $[a, b]$, the graph of $f(x)$ is below the line connecting $(a, f(a)$ and $(b, f(b))$. See the hint of Homework 8 for an illustration.) Prove that if a function $f : \mathbb{R} \to \mathbb{R}$ is convex, then the graph $y = f(s)$ is always above any of its tangent line. That is, if $P(x)$ is the equation of tangent line to $y = f(x)$ at $x = a$, then $f(x) \geq P(x)$ for all x.

2. Rudin Chapter 5, Problem 14. (You might find Problem 1 useful.)

3. Rudin Chapter 5, Problem 15. (Don't worry about the "To show..." at the end of the problem.)

4. Rudin Chapter 5, Problem 17. (Hint: first show that $P_0 = P_1 = 0$, and therefore $P_2 = ax^2$.)

5. Rudin Chapter 5, Problem 25 part (a) - part (c). (For part (b), the result follows easily from (a) if we know that x_n is convergent, why? For part (c), apply Taylor's Theorem at x_n. You might find Problem 2 might be useful.)

6. Rudin Chapter 5, Problem 26. (Start with $|f(x)| = |f(x) - f(a)| = \cdots$.)

7. Salas 4.11: 17, 20.

8. Salas 11:5: 10, 28, 48.

9. Salas 11.6: 15, 28, 42.

10. Salas 12.6: 12, 16, 18, 22, 32.

11. Salas 12.7: 10, 22, 29.