Names and Student IDs: _____

Homework 6 Calculus 1

1. The function e^x has two identical definitions (shown in class):

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n.$$

Without using differentiation, do the followings:

(a) Show the series above converges absolutely for all x.

(b) Using either definition, show that for all $x, y \in \mathbb{R}, e^{x+y} = e^x e^y$.

(c) Show that for all $x \in \mathbb{R}, e^{-x} = \frac{1}{e^x}$, and therefore $e^{x-y} = \frac{e^x}{e^y}$.

(d) Show that $e^x > 0$ for all $x \in \mathbb{R}$.

(e) Show that for all $x, y \in \mathbb{R}, (e^x)^y = e^{xy}$. (Note that you can't raise $1 + \frac{x}{n}$ above to the power ny, since that is what we are proving here.

(f) Show that e^x is strictly increasing. That is, $x > y \Rightarrow e^x > e^y$. (It might be useful to first show that $e^x > 1 \; \forall x > 0$.)

Note that you are *NOT* allowed to use any "power rule" you have learnt before. You need to derive them.

2. Salas 12.5: 10, 18, 20, 35, 42.

3. Salas 12.8: 8, 20, 30, 35.