Names and Student IDs: ____

Homework 7 Calculus 1

1. Rudin Chapter 2, Problem 9 (a) - (c), and the following statement:

$$E^0(\text{ or } int(E)) = \bigcup_{G \subset E,\, G\ open} G.$$

   The problem says that E^0 is the *largest* open subset contained in E. (Compare with Theorem 2.27, which says that the closure $\bar{E}$ of E is the *smallest* closed set containing E).

2. Prove that
   (a) $(0,1) \subset \mathbb{R}$ is not compact. (Note: open $\neq$ not closed.)
   (b) $\{\frac{1}{n}\}_{n=1}^{\infty} \subset \mathbb{R}$ is not compact.
   (c) $\{\frac{1}{n}\}_{n=1}^{\infty} \cup \{0\} \subset \mathbb{R}$ is compact. It is called the *compactification* of (b).

3. Rudin Chapter 4, Problem 1.

   For the following two problems, for $f : A \to B$, define

$$for\ E \subset A, f(E) := \{f(x) \mid x \in E\}.$$

   and

$$for\ V \subset B, f^{-1}(V) := \{x \in E \mid f(x) \in V\}.$$

4. Prove that the statement

$$\text{for all closed subset } C \subset \mathbb{R},\ f^{-1}(C) \text{ is closed in } \mathbb{R}$$

   is equivalent to

$$\text{for all open subset } E \subset \mathbb{R},\ f^{-1}(E) \text{ is open in } \mathbb{R}$$

   It turns out that both statements are equivalent the fact that $f : \mathbb{R} \to \mathbb{R}$ is continuous (will be shown in class).

5. Rudin, Chapter 4, Problem 3 with $X = \mathbb{R}$. (It might be useful to prove that $\{a\} \subset \mathbb{R}$ is a closed set $\forall a \in \mathbb{R}$.

6. Given $f : [0,1] \to \mathbb{R}$ continuous, and suppose that $f([0,1]) \subset \mathbb{Q}$. Prove that if $f(\frac{1}{2}) = 0$, then f is a constant function.

7. Salas 2.4: 6, 14, 26, 29, 34.

8. Salas 2.6: 5, 9, 10, 15, 25, 26, 28.