Homework 8 Calculus 1

1. Prove that $f : A \to B$ is continuous if and only if $f^{-1}((a, b))$ is open in A, for every open interval $(a, b) \subset B$.

2. If $f : \mathbb{R} \to \mathbb{R}$ satisfies the equation

$$f(x + y) = f(x) + f(y)$$

   for all $x, y \in \mathbb{R}$, and $f(1) = 1$,

   (a) Find the values of $f(x)$ for all $x \in \mathbb{Q}$.

   (b) If in addition f is continuous, show that $f(x) = x$.

3. Rudin Chapter 4, Problem 15. (See Definition 4.28 for monotonic functions.)

4. Rudin Chapter 4, Problem 23 (Just prove the first statement).

5. Prove that if $f(x)$ is monotonic on $[a, b]$ and satisfies the conclusion of intermediate value theorem, then $f(x)$ is continuous.

6. Rudin Chapter 5, Problem 1. (You may assume the fact that $f' \equiv 0 \Rightarrow f$ is constant.)

7. Salas 3.1: 9, 14, 18, 35, 45, 52, 59.