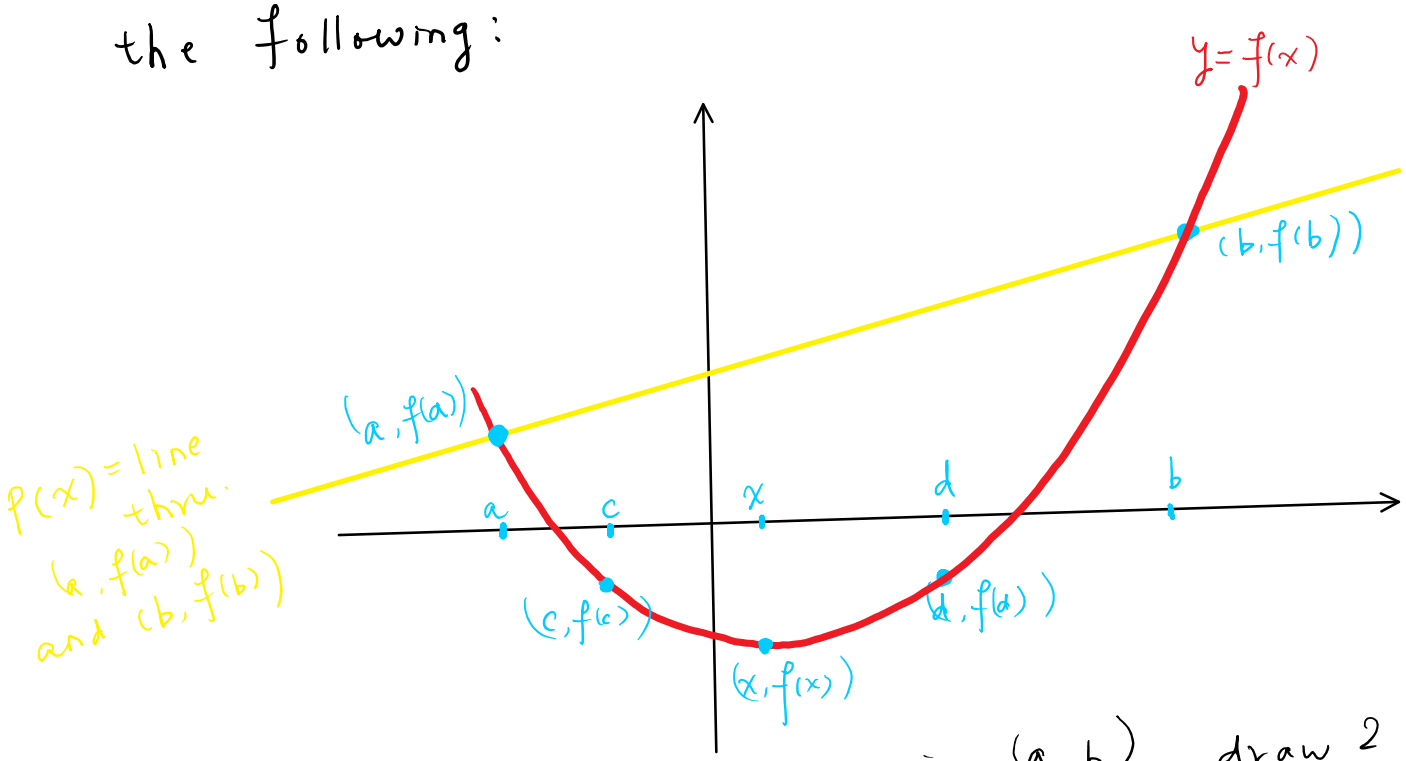
Homework 8 Hints

2016年11月10日 17:14

#4: If f is convex, then f is continuous.

- An easy way to view a convex function is the following:



For any 3 points $c < x < d$ in (a, b), draw 2 lines connecting $(c, f(c))$ to $(x, f(x))$ and $(x, f(x))$ to $(d, f(d))$

Call the two functions $g(x)$ and $h(x)$, what can you say about $g(x)$, $h(x)$, and $f(x)$?

- Use the relation you just wrote down to conclude that $\lim_{y \to x} f(y) = f(x)$.

and hence f is continuous at x

- For an extra credit of 10 points, prove the relation you wrote down in general.

#5 For simplicity, you may assume that the function is strictly monotonic. that is

$x < y \Rightarrow f(x) < f(y)$ ← strictly increasing

or $x < y \Rightarrow f(x) > f(y)$ ← " decreasing.

To prove continuity, you might want to apply problem 1:

take $(\alpha, \beta) \subset$ range of f

$\Rightarrow \alpha = f(a), \beta = f(b)$

Try to prove that

$$f^{-1}((\alpha,\beta)) = \underset{\text{if } a<b}{(a,b)} \text{ or } \underset{\text{if } a>b}{(b,a)}$$

#3. Try to show that if f is not monotonic, then $\exists\, x$ s.t. near x, f looks like ∩ or ∪

Derive a contradiction.