Name and Student ID's: ____________________

Homework 9, Advanced Calculus 1

A topological space X is *sequentially compact* if every sequence has a convergent subsequence.

A compact space is sequentially compact (cf. Theorem 2.37 of Rudin, which is valid for general topological spaces). The first two problems below will prove that for metric space, the two compactness properties are actually equivalent.

1. Prove that for a sequentially compact metric space (X, d), every open cover has a *countable* subcover.
2. Use Problem 1 to show that (X, d) is compact.
3. Rudin Chapter 7 Exercise 8
4. Rudin Chapter 7 Exercise 9
5. Rudin Chapter 7 Exercise 10
6. Rudin Chapter 7 Exercise 15
7. Rudin Chapter 7 Exercise 16
8. Rudin Chapter 7 Exercise 11
9. Rudin Chapter 7 Exercise 12
10. Rudin Chapter 7 Exercise 13
11. Prove Theorem 7.17, with additional assumption that f_n' is continuous for all n.