Name and Student ID's: ____

Homework 1, Undergraduate Analysis 1

1. Fill in the gaps in the construction of the non-measurable space in class. (The equivalence relation, and the partition of $[0,1)$ into translations of N.)

2. Prove that any intersection of a family of $\sigma-$algebras is still a $\sigma-$algebra.

3. Rudin Chapter 11, Exercise 5.

4. Rudin Chapter 11, Exercise 6.

5. Given a set X, a family of subsets $\mathcal{R} \subset \mathcal{P}(X)$ is called a *ring* if it is closed under finite unions and complements:

   - $E, F \in \mathcal{R} \Rightarrow E \backslash F \in \mathcal{R}$.
   - $E_1, \ldots, E_n \in \mathcal{R} \Rightarrow \cup_{i=1}^{n} E_i \in \mathcal{R}$.

   A ring is called σ-ring if is closed under countable union:

   - $\{E_i\}_{i=1}^{\infty} \subset \mathcal{R} \Rightarrow \cup_{i=1}^{\infty} E_i \in \mathcal{R}$.

   Prove that a σ-ring $\mathcal{R}$ is a σ-algebra if and only if $X \in \mathcal{R}$.

6. Let $\mathcal{M}$ be an infinite $\sigma-$algebra,
   (a) Prove that $\mathcal{M}$ has an infinite sequence of non-empty disjoint sets.
   (b) Prove that $\mathcal{M}$ is uncountable.

7. Prove that an algebra $\mathcal{A}$ is a σ-algebra if it is closed under countable *increasing* unions:

$$\text{If } \{E_i\}_{i=1}^{\infty} \subset \mathcal{A} \text{ and } E_i \subset E_{i+1} \text{ for all } i, \text{ then } \cup_{i=1}^{\infty} E_i \in \mathcal{A}.$$