Name and Student ID's: ____

Homework 3, Undergraduate Analysis

Refer to class notes for definitions of sets and functions in this homework assignment. All functions and sets are measurable.

1. If $f \geq 0$ and $\int_E f \, d\mu = 0$, then $f = 0$ a.e. on E.
2. Prove the following basic properties for Lebesgue integrations:
   (a) $f(x) \in [a, b]$ for all $x \in X$, $\mu(X) < \infty$, then
   $$a\mu(X) \leq \int_X f \, d\mu \leq b\mu(X).$$
   (b) $f \leq g \Rightarrow \int_X f \, d\mu \leq \int_X g \, d\mu$.
   (c) $\forall c \in \mathbb{R}$, $\int_E cf \, d\mu = c \int_E f \, d\mu$.
   (d) $\int_E f \, d\mu = \int_X f K_E \, d\mu$.
   (e) $f \in \mathcal{L}(X) \Rightarrow f \in \mathcal{L}(E)$ for all measurable subset E of X.
3. Rudin Chapter 11, Exercise 8.
4. Rudin Chapter 11, Exercise 12.
5. Rudin Chapter 11, Exercise 11.
6. Rudin Chapter 11, Exercise 16.
7. If $f \geq 0$ integrable on X, then for all $\epsilon > 0$, there exists $E \in \mathcal{M}$ such that $\mu(E) < \infty$ and
   $$\int_E f \, d\mu > \int_X f \, d\mu - \epsilon.$$