Hw 10: §10-1 6, 10, 16, 22, 26

§15-1: 6, 14

§15-2: 4, 12, 22, 26, 30, 36, 42

§15-3: 8, 18, 22, 24, 28, 30.

§10-1

※ 6.

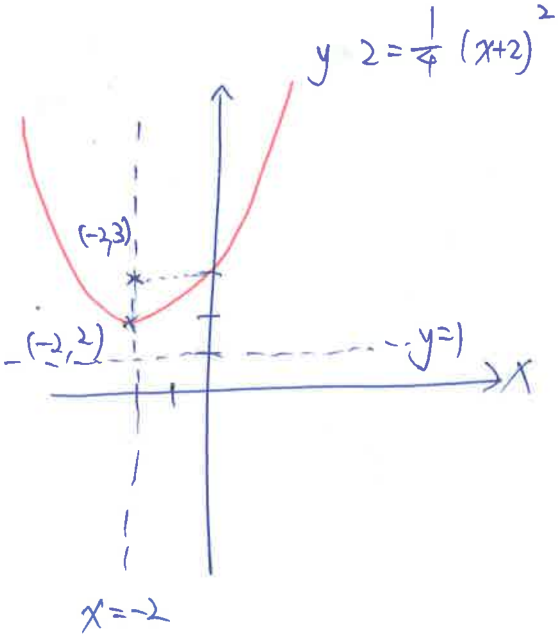
$y-2=\frac{1}{4}(x+2)^2$

vertex $(-2,2)$

focus $(-2,3)$

axis $x=-2$

directrix $y=1$



※ 10.

$\frac{x^2}{4}+\frac{y^2}{9}=1$

center $(0,0)$

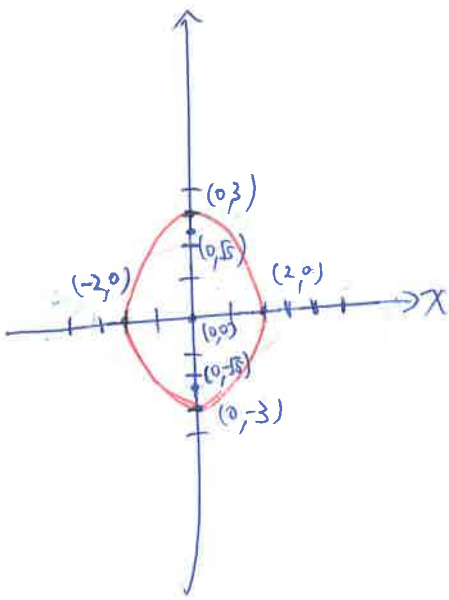
foci $(0,\pm\sqrt{5})$

$\because a^2=9,\ b^2=4$

$\therefore c^2=5$

length of major axis $=2a=6$

length of minor axis $=2b=4$



16.

$$16(x-2)^2+25(y-3)^2=400$$

$$\frac{(x-2)^2}{25}+\frac{(y-3)^2}{16}=1$$

∵ $a^2=25$, $b^2=16$

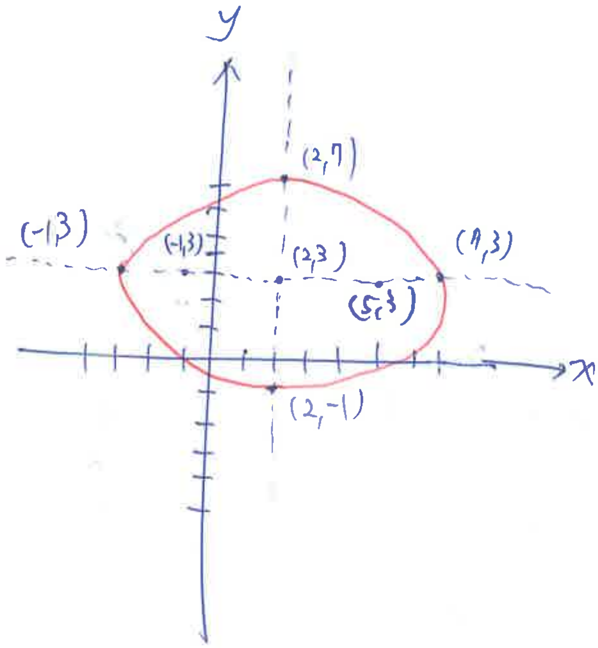
∴ $c^2=9$

center $(2,3)$

foci $(5,3)$, $(-1,3)$

length of major axis $=2a=10$

length of minor axis $=2b=8$



22.

$$\frac{y^2}{9}-\frac{x^2}{16}=1$$

∵ $a^2=9$, $b^2=16$

∴ $c^2=25$

center $(0,0)$

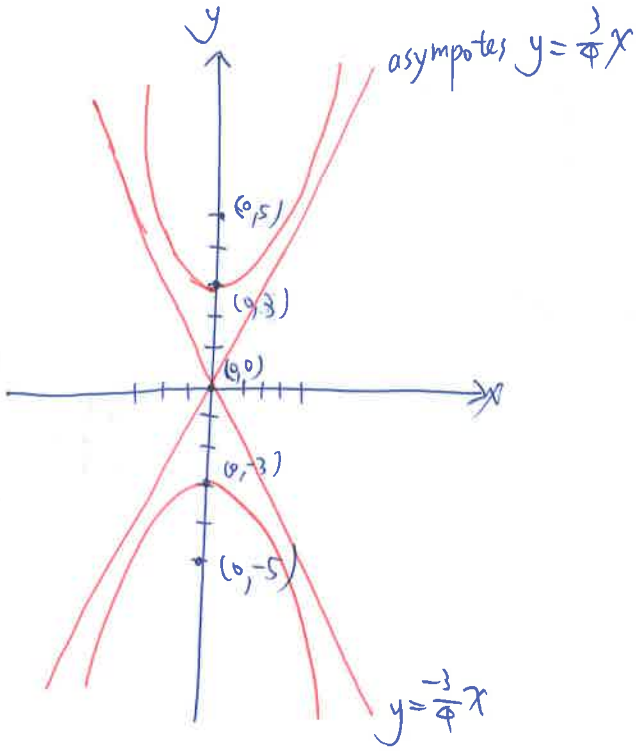
transverse axis $=2a=6$

vertices $(0,\pm3)$

foci $(0,\pm5)$

asymptotes $y=\frac{3}{4}x$

$y=-\frac{3}{4}x$



26.

$$-3x^2+y^2-6x=0$$

$$y^2-3(x^2+2x)=0$$

$$y^2-3(x+1)^2=-3$$

$$\frac{(x+1)^2}{1}-\frac{y^2}{3}=1$$

$\therefore\ a^2=1 \quad b^2=3$

$\therefore\ c^2=4$

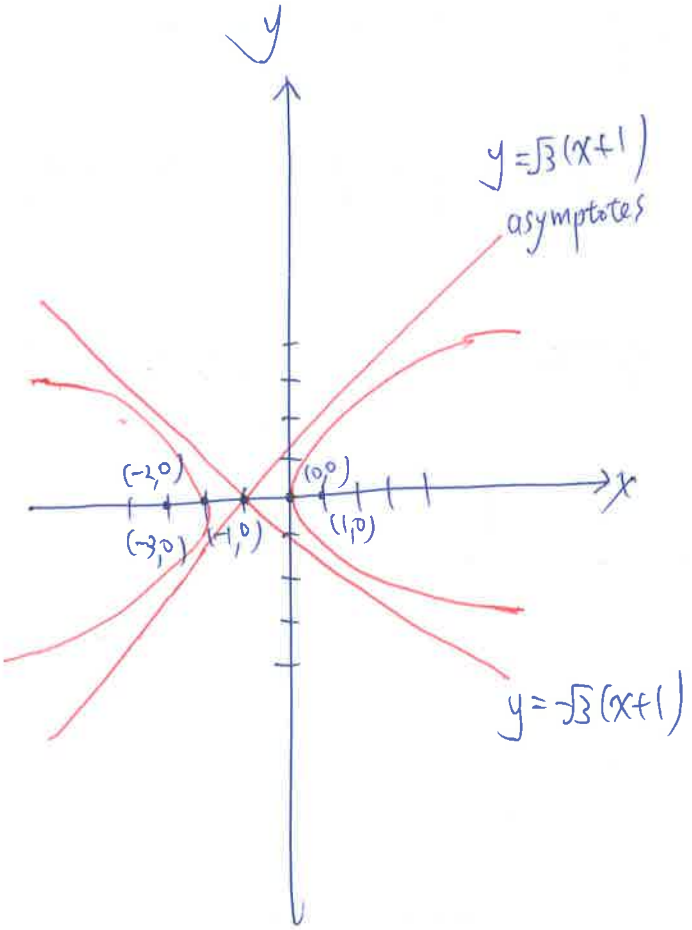
center $(-1,0)$

transverse axis $=2a=2$

vertices $(0,0)$, $(-2,0)$

foci $(1,0)$, $(-3,0)$

asymptotes $y=\pm\sqrt{3}(x+1)$



§15-1

※6.

$$f(x,y)=\frac{x^2}{x^2+y^2}$$

as $x^2+y^2 \neq 0$, $f(x,y)$ is well-defined

domain $= \{(x,y) \mid x^2+y^2 \neq 0,\ x\in\mathbb{R},\ y\in\mathbb{R}\}$

since $x^2+y^2 \geq x^2$, then $\frac{x^2}{x^2+y^2} \leq 1$ and $\frac{x^2}{x^2+y^2} \geq 0$

range $= [0,1]$

※14.

$$f(x,y,z)=\frac{z^2}{x^2-y^2}$$

as $x^2-y^2 \neq 0$, $x^2 \neq y^2$, $f(x,y,z)$ is well-defined.

domain $= \{(x,y) \mid x^2 \neq y^2,\ x\in\mathbb{R},\ y\in\mathbb{R}\}$

range $= (-\infty, \infty)$

§15-2

※4

$$x^2-4y^2-2z=0 \Rightarrow z=\frac{x^2}{2}-2y^2=\frac{x^2}{2}-\frac{y^2}{\frac{1}{2}}$$

∴ a hyperbolic paraboloid

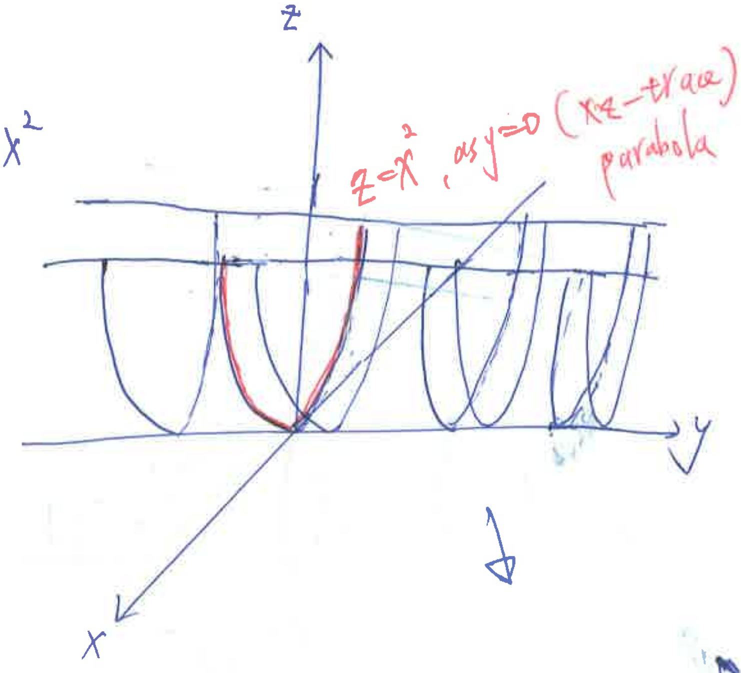
※12.

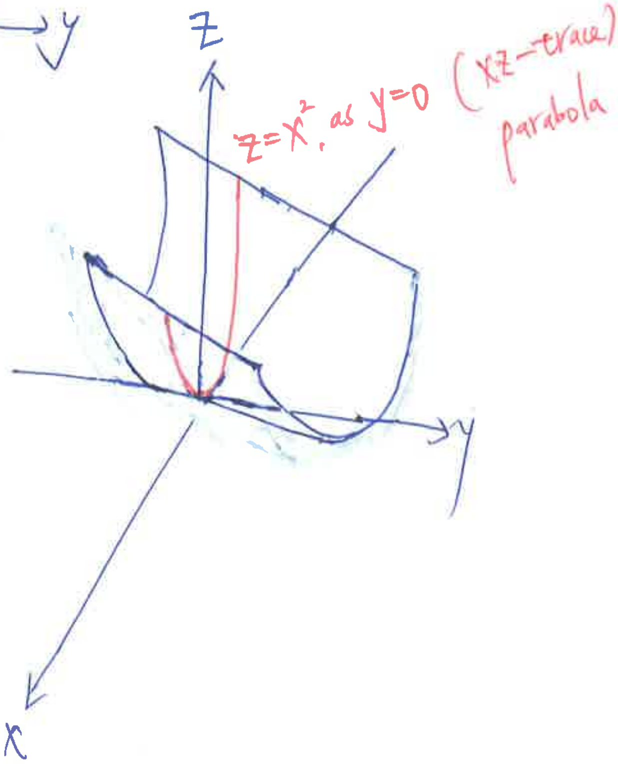
$$x-y^2-6z^2=0 \Rightarrow x=y^2+6z^2=\frac{y^2}{1}+\frac{z^2}{\frac{1}{6}}$$

∴ an elliptic paraboloid

※22.

$z=x^2$



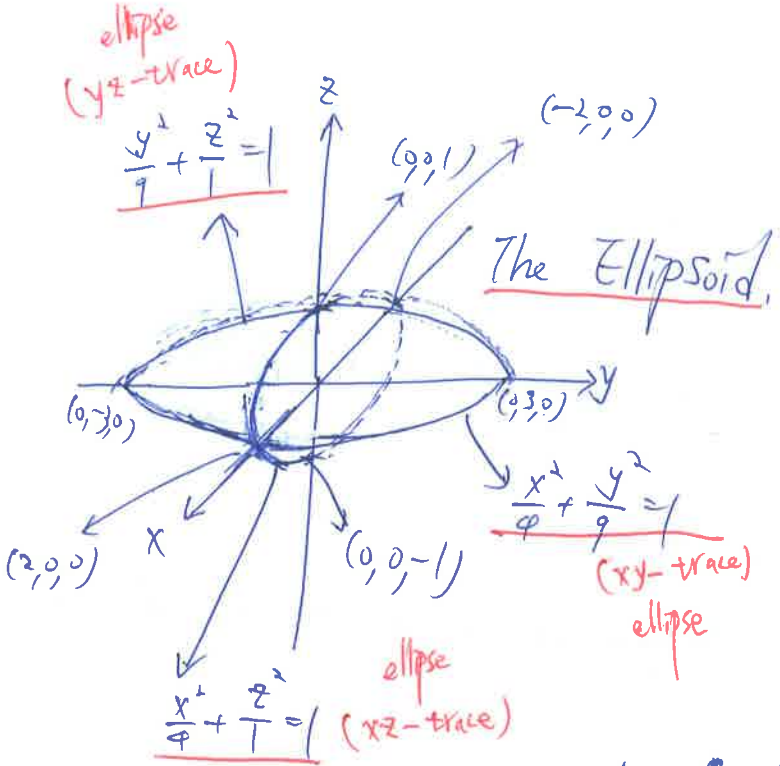


#26.

$$9x^2+4y^2+36z^2-36=0$$

$$\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{1}=1$$

$$a^2=4 \quad b^2=9 \quad c^2=1$$



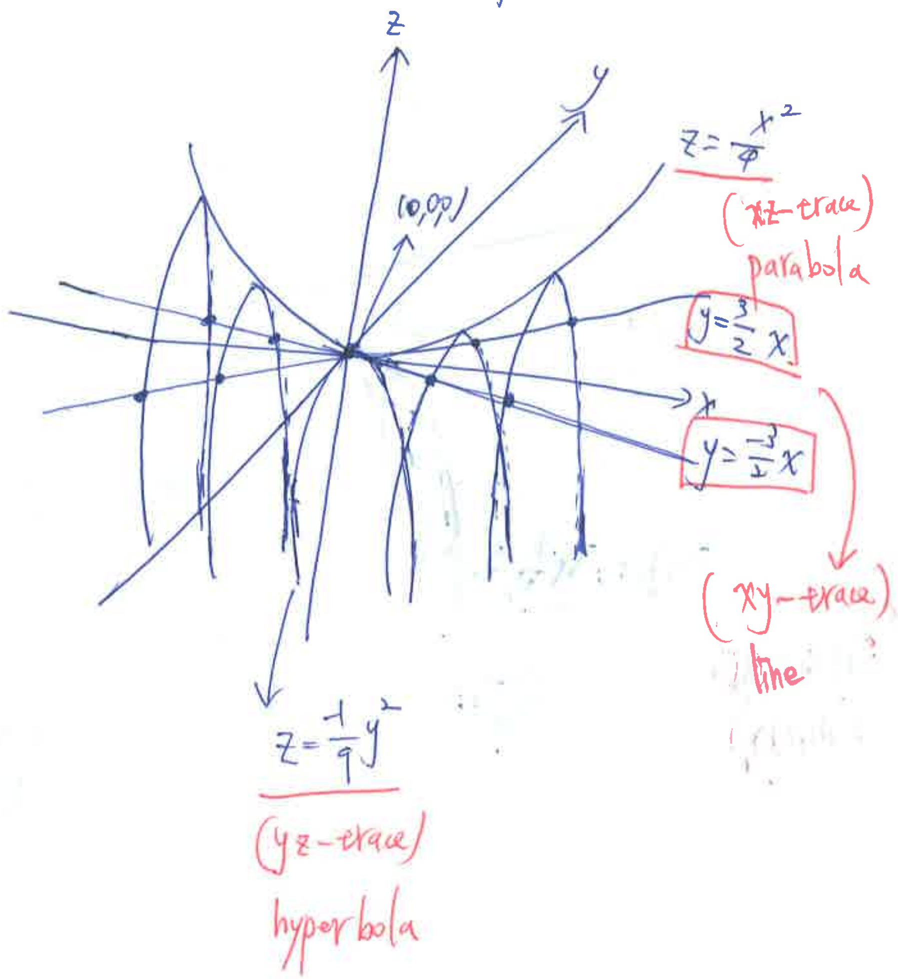
The Hyperbolic Paraboloid

#30.

$$9x^2-4y^2-36z=0$$

$$z=\frac{x^2}{4}-\frac{y^2}{9}$$

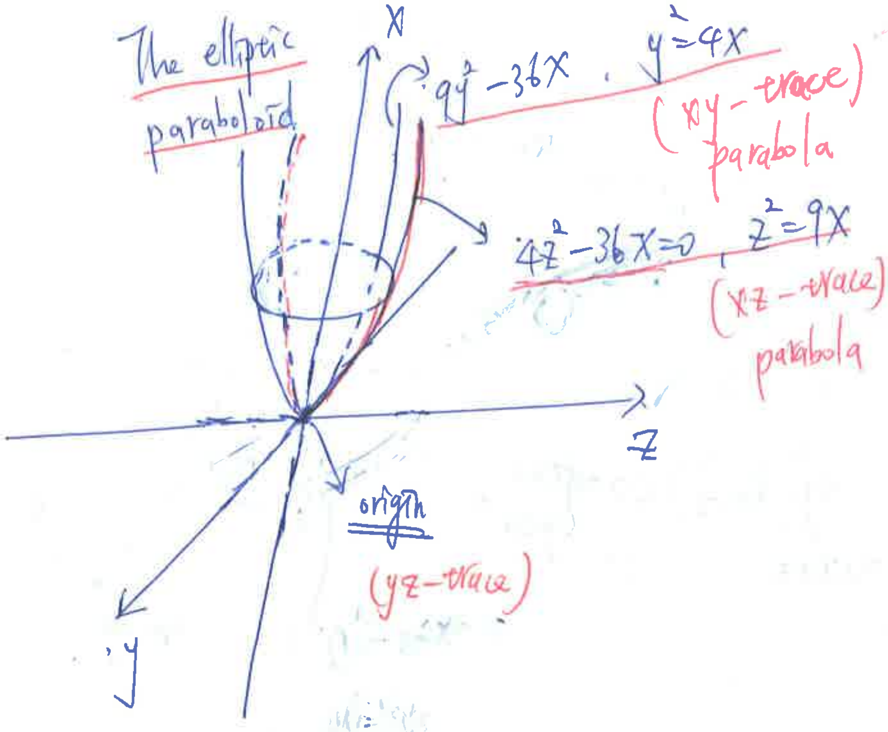
$$a^2=4, \quad b^2=9$$



#36 4

$9y^2+4z^2-36x=0$

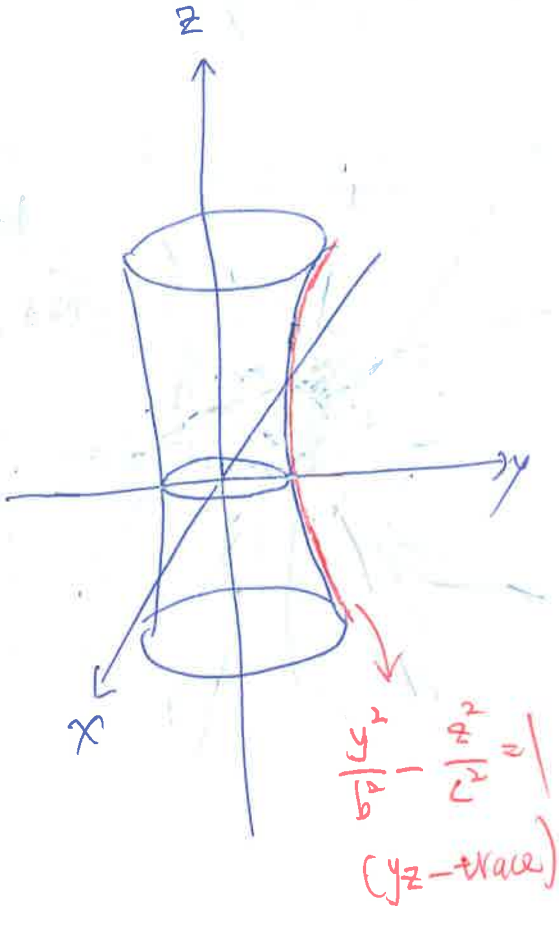
$$x=\frac{y^2}{4}+\frac{z^2}{9}$$



#42.

The hyperbola $c^2y^2-b^2z^2-b^2c^2=0$ is revolved about the z-axis.

$$\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \quad \text{(one sheet)}$$



§15-3

※ 8. $f(x,y) = e^{xy}$, $c = \frac{1}{2}, 1, 2, 3$ → the coordinate axes and the hyperbolas $xy = \ln c$.

① $e^{xy} = \frac{1}{2}$

$xy = \ln(\frac{1}{2}) < 0$

$\Rightarrow \begin{cases} x>0 \\ y<0 \end{cases}$ or $\begin{cases} x<0 \\ y>0 \end{cases}$

② $e^{xy} = 1$

$xy = \ln 1 = 0$

$\Rightarrow x=0$ or $y=0$

(y-axis) (x-axis)

③ $e^{xy} = 2$

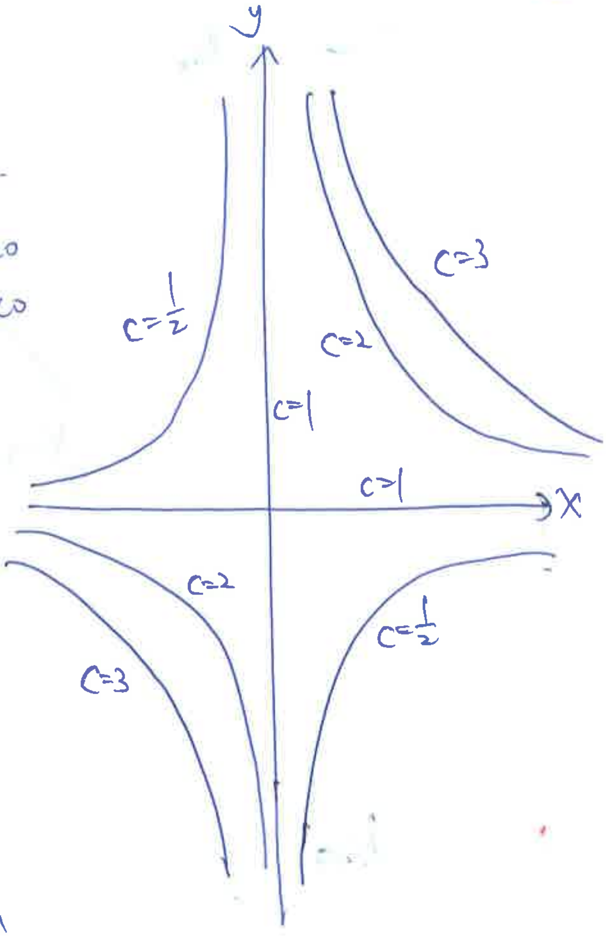
$xy = \ln 2 > 0$

$\begin{cases} x>0 \\ y>0 \end{cases}$ or $\begin{cases} x<0 \\ y<0 \end{cases}$

④ $e^{xy} = 3$

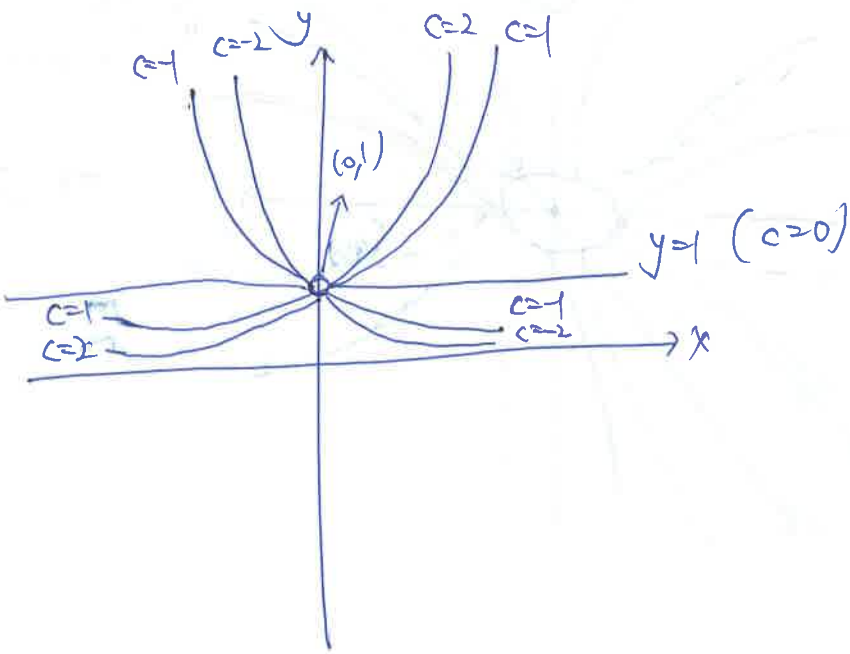
$xy = \ln 3 > 0$

$\Rightarrow \begin{cases} x>0 \\ y>0 \end{cases}$ or $\begin{cases} x<0 \\ y<0 \end{cases}$



※ 18. $f(x,y) = \frac{\ln y}{x}$, $c = -2, -1, 0, 1, 2$,

$\frac{\ln y}{x} = c \Rightarrow \ln y = cx \Rightarrow y = e^{cx}$ with the point $(0,1)$ omitted.
(since $x \neq 0$)



\# 22.

$f(x,y,z) = \frac{1}{4}x^2 + \frac{1}{6}y^2 + \frac{1}{9}z^2$, $c=1$

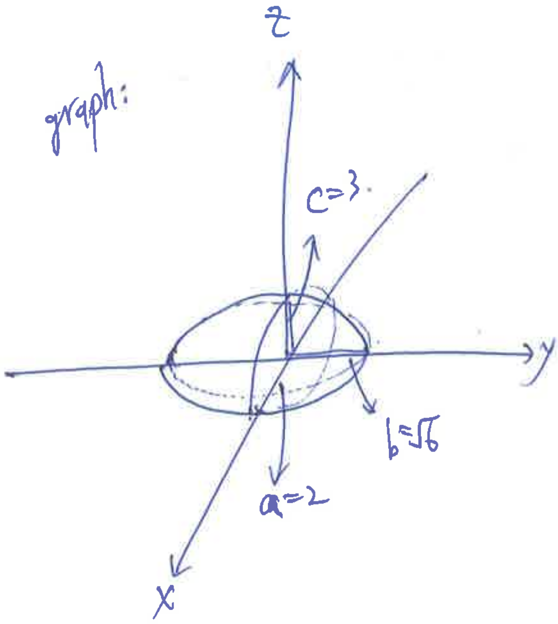
$\Rightarrow \frac{x^2}{4} + \frac{y^2}{6} + \frac{z^2}{9} = 1$ (ellipsoid)

$a^2 = 2$

$b^2 = 6$

$c^2 = 9$

graph:



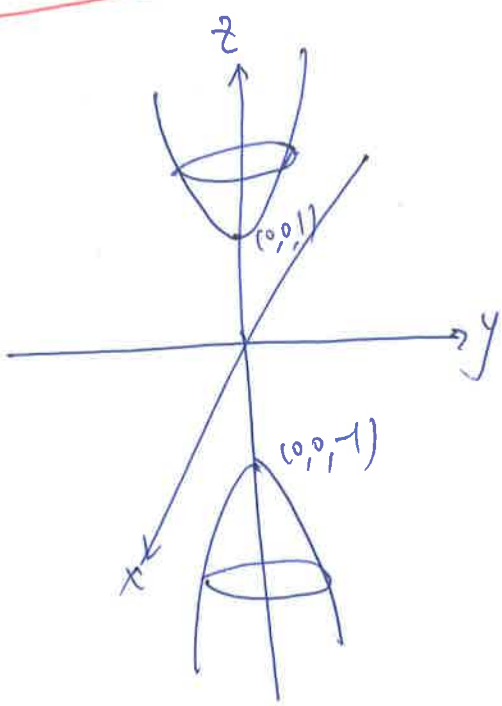
\# 24.

$f(x,y,z) = z^2 - 36x^2 - 9y^2$, $c=1$

$\Rightarrow z^2 - 36x^2 - 9y^2 = 1$

$\Rightarrow \frac{x^2}{\left(\frac{1}{6}\right)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} - \frac{z^2}{1} = -1$ (hyperboloid of two sheets)

$c^2 = 1$, $c = \pm 1$



#28.

$f(x,y) = (x^2+y^2)\cdot e^{xy}$, $P(1,0)$

$\because f(1,0) = 1\cdot e^0 = 1$

$\therefore (x^2+y^2)\cdot e^{xy} = 1$ (level curve contains the point p)

#30

$f(x,y) = (x^2+y)\cdot \ln[2-x+e^y]$, $P(2,1)$

$\because f(2,1) = 5\cdot \ln(e) = 5$

$\therefore (x^2+y)\cdot \ln[2-x+e^y] = 5$ (level curve contains the point p)