Hw 11 :

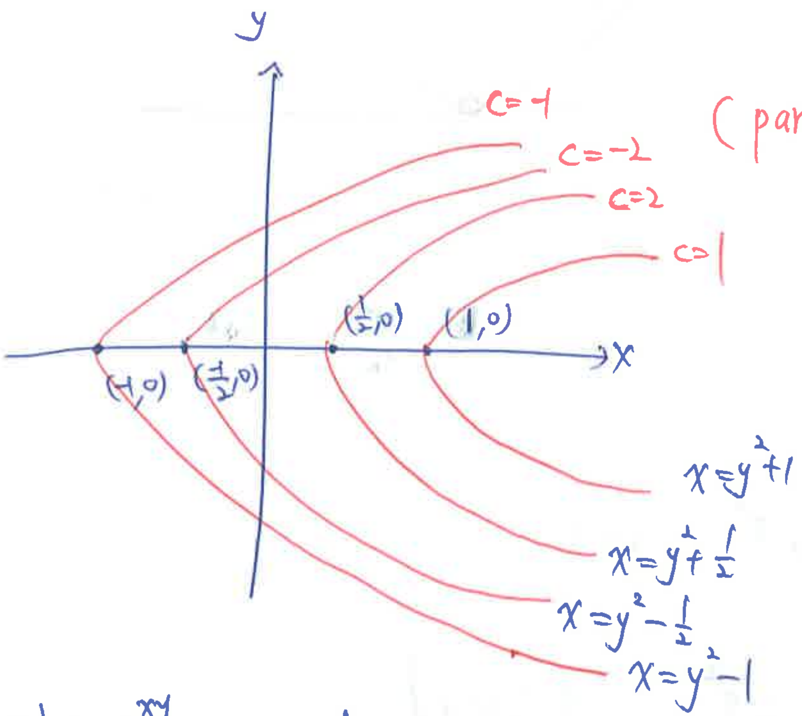
§15-3 : 4, 8, 14, 22, 26, 30

§15-4 : 6, 10, 14, 20, 28, 32, 38, 42, 52, 56.

15-3

#4. $f(x,y)=\frac{1}{x-y^2}$ $c=-2,-1,1,2$

$\frac{1}{x-y^2}=c \Rightarrow x-y^2=\frac{1}{c} \Rightarrow x=y^2+\frac{1}{c}$ ∴ vertex $(\frac{1}{c},0)$



#8 $f(x,y)=e^{xy}$, $c=\frac{1}{2},1,2,3.$

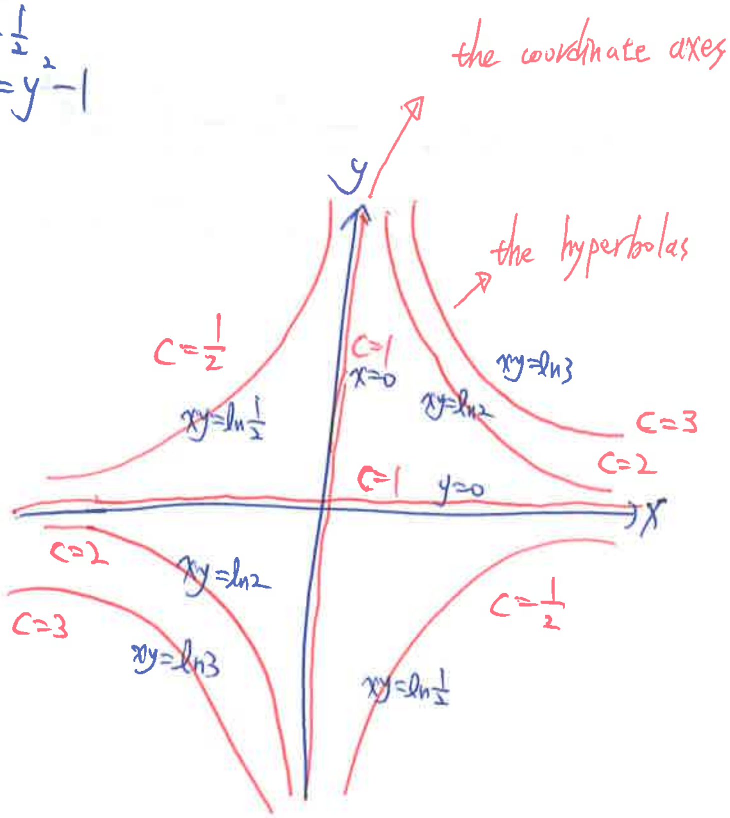
$e^{xy}=c \Rightarrow xy=\ln c$

$c=1 \Rightarrow xy=\ln 1=0 \Rightarrow x=0$ or $y=0$

$c=2 \Rightarrow xy=\ln 2>0$

$c=3 \Rightarrow xy=\ln 3>0$

$c=\frac{1}{2} \Rightarrow xy=\ln\frac{1}{2}=-\ln 2<0.$



14.

$f(x,y) = \ln\left(\frac{y}{x^2}\right)$ $c = -2, -1, 0, 1, 2$

$\Rightarrow \ln\left(\frac{y}{x^2}\right) = c \Rightarrow \frac{y}{x^2} = e^c \Rightarrow y = e^c \cdot x^2$ and $x \neq 0$ (parabolas)

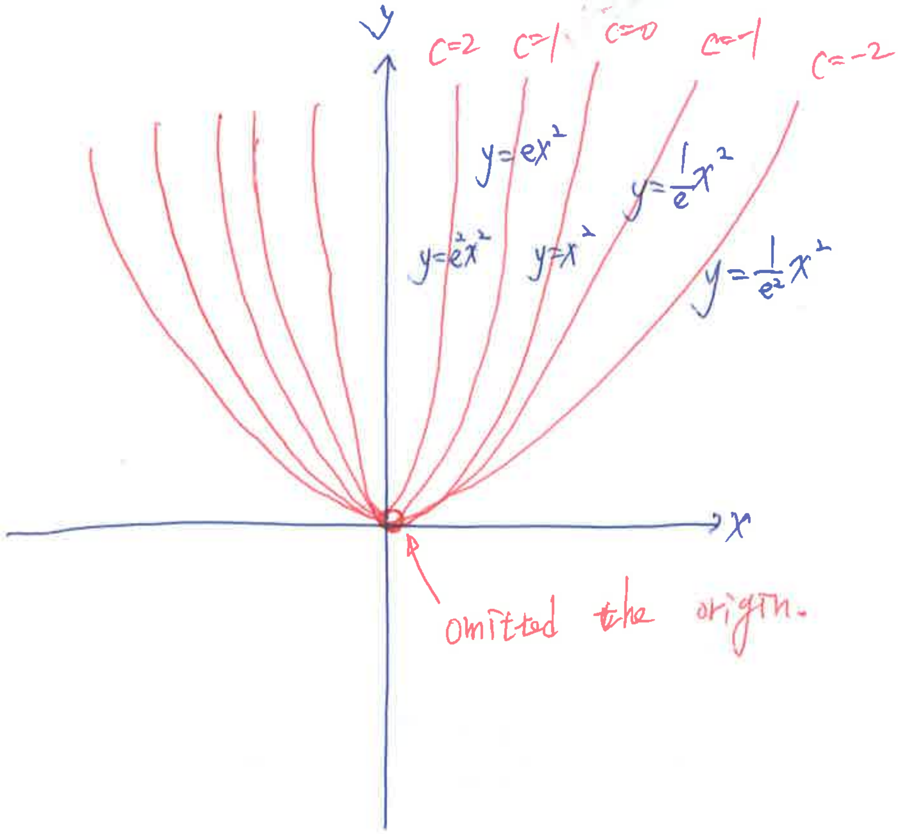
$c = -2 \Rightarrow y = \frac{1}{e^2} x^2$

$c = -1 \Rightarrow y = \frac{1}{e} x^2$

$c = 0 \Rightarrow y = x^2$

$c = 1 \Rightarrow y = e x^2$

$c = 2 \Rightarrow y = e^2 x^2$

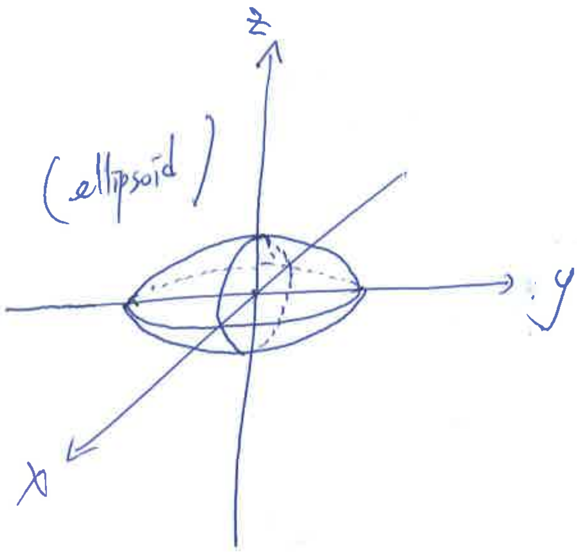


22.

$f(x,y,z) = \frac{1}{4}x^2 + \frac{1}{6}y^2 + \frac{1}{9}z^2$, $c = 1$

c-level surface : $\frac{1}{4}x^2 + \frac{1}{6}y^2 + \frac{1}{9}z^2 = 1$ (ellipsoid)

$a = 2$ $b = \sqrt{6}$ $c = 3$



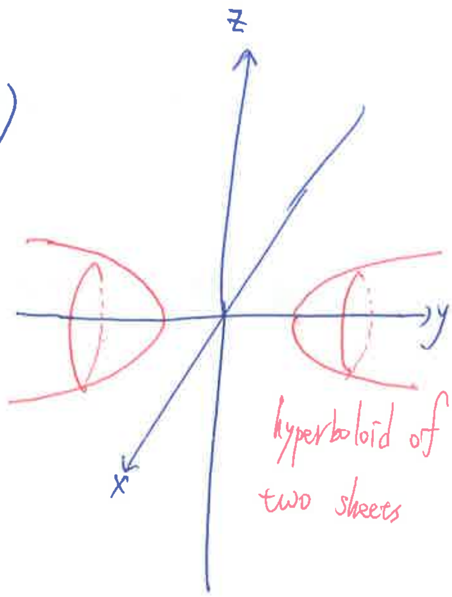
26.

$f(x,y,z) = 9x^2 - 4y^2 + 36z^2$

① $c < 0$

$9x^2 - 4y^2 + 36z^2 < 0$ (hyperboloid of two sheets)

$9x^2 + 36z^2 < 4y^2$



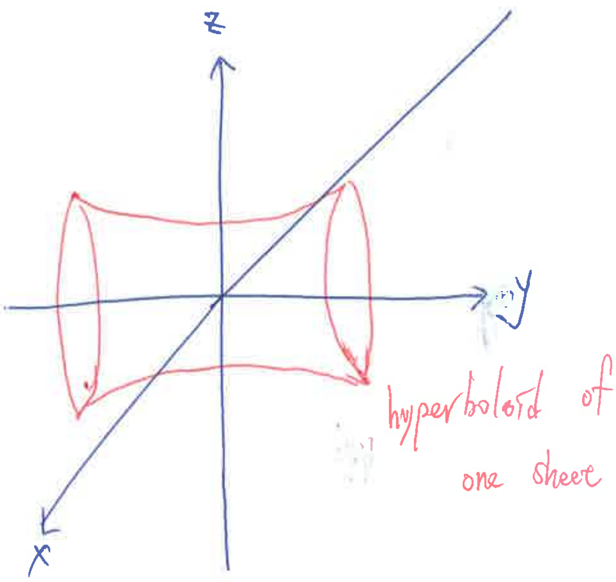
② $c = 0$

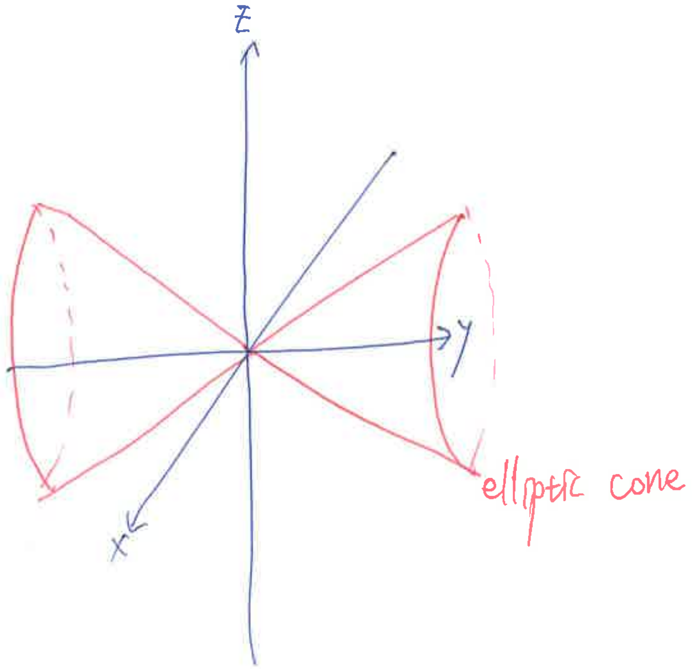
$9x^2 - 4y^2 + 36z^2 = 0$ (elliptic cone)

③ $c > 0$

$9x^2 - 4y^2 + 36z^2 > 0$ (hyperboloid of one sheets)

$9x^2 + 36z^2 > 4y^2$





30. $f(x,y) = (x^2+y)\cdot\ln[2-x+e^y]$, $P(2,1)$

$f(2,1) = (4+1)\cdot\ln[2-2+e^1] = 5$

$\therefore (x^2+y)\cdot\ln[2-x+e^y] = 5$

§15-4.

\#6. $z = \sqrt{x^2-3y} = (x^2-3y)^{\frac{1}{2}}$

$$\frac{\partial z}{\partial x} = \frac{1}{2}\cdot(x^2-3y)^{-\frac{1}{2}}\cdot(2x) = x\cdot(x^2-3y)^{-\frac{1}{2}} = \frac{x}{\sqrt{x^2-3y}}$$

$\therefore$

$$\frac{\partial z}{\partial y} = \frac{1}{2}\cdot(x^2-3y)^{-\frac{1}{2}}\cdot(-3) = \frac{-3}{2\sqrt{x^2-3y}}$$

※10. $z = Ax^2+Bxy+Cy^2$

$$\frac{\partial z}{\partial x} = 2Ax+By$$

$$\therefore \frac{\partial z}{\partial y} = Bx+2Cy$$

※14. $f(x,y) = (x+y)\cdot\sin(x-y)$

$$\frac{\partial f}{\partial x} = \frac{\partial(x+y)}{\partial x}\cdot\sin(x-y) + (x+y)\cdot\frac{\partial(\sin(x-y))}{\partial x}$$

$$\frac{\partial f}{\partial x} = \sin(x-y) + (x+y)\cdot\cos(x-y)\cdot 1 = \sin(x-y)+(x+y)\cdot\cos(x-y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial(x+y)}{\partial y}\cdot\sin(x-y) + (x+y)\cdot\frac{\partial \sin(x-y)}{\partial y}$$

$$\frac{\partial f}{\partial y} = \sin(x-y) + (x+y)\cdot\cos(x-y)\cdot(-1) = \sin(x-y)-(x+y)\cdot\cos(x-y)$$

20. $f(x,y,z) = e^{xy} \cdot \sin(xz)$

$$\frac{\partial f}{\partial x} = \frac{\partial(e^{xy})}{\partial x} \cdot \sin(xz) + e^{xy} \cdot \frac{\partial(\sin(xz))}{\partial x}$$

$$= e^{xy} \cdot y \cdot \sin(xz) + e^{xy} \cdot \cos(xz) \cdot z$$

$$= e^{xy}(y \sin(xz) + z\cos(xz))$$

$$\frac{\partial f}{\partial y} = \frac{\partial(e^{xy})}{\partial y} \cdot \sin(xz) + e^{xy} \cdot \frac{\partial(\sin(xz))}{\partial y}$$

$$= e^{xy} \cdot x \cdot \sin(xz) + e^{xy} \cdot 0$$

$$= x \cdot e^{xy} \cdot \sin(xz)$$

$$\frac{\partial f}{\partial z} = \frac{\partial(e^{xy})}{\partial z} \cdot \sin(xz) + e^{xy} \cdot \frac{\partial(\sin(xz))}{\partial z}$$

$$= 0 \cdot \sin(xz) + e^{xy} \cdot \cos(xz) \cdot x$$

$$= x \cdot e^{xy} \cdot \cos(xz)$$

※28.

$$w = xy\cdot\sin z - yz\cdot\sin x$$

$$\frac{\partial w}{\partial x} = \frac{\partial(xy\sin z)}{\partial x} - \frac{\partial(yz\sin x)}{\partial x}$$

$$= y\sin z - yz\cdot\cos x$$

$$\frac{\partial w}{\partial y} = \frac{\partial(xy\sin z)}{\partial y} - \frac{\partial(yz\sin x)}{\partial y}$$

$$= x\sin z - z\sin x$$

$$\frac{\partial w}{\partial z} = \frac{\partial(xy\sin z)}{\partial z} - \frac{\partial(yz\sin x)}{\partial z}$$

$$= xy\cdot\cos z - y\sin x$$

※32.

$$g(x,y) = \frac{x}{x+y^2}$$

$$g_x = \frac{\frac{\partial x}{\partial x}\cdot(x+y^2) - x\cdot\frac{\partial(x+y^2)}{\partial x}}{(x+y^2)^2} = \frac{1\cdot(x+y^2) - x\cdot 1}{(x+y^2)^2} = \frac{y^2}{(x+y^2)^2}$$

$$g_x(1,2) = \frac{4}{25}$$

$$g_y = \frac{\frac{\partial x}{\partial y}\cdot(x+y^2) - x\cdot\frac{\partial(x+y^2)}{\partial y}}{(x+y^2)^2} = \frac{0\cdot(x+y^2) - x\cdot 2y}{(x+y^2)^2} = \frac{-2xy}{(x+y^2)^2}$$

$$g_y(1,2) = \frac{-4}{25}$$

38.

$$f(x,y)=e^{2x+3y}$$

$$\frac{\partial f}{\partial x} = e^{2x+3y}\cdot \frac{\partial(2x+3y)}{\partial x} = 2e^{2x+3y}$$

$$\frac{\partial f}{\partial y} = e^{2x+3y}\cdot \frac{\partial(2x+3y)}{\partial y} = 3\cdot e^{2x+3y}$$

$$\frac{\partial f}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h} = \lim_{h\to 0}\frac{e^{2(x+h)+3y}-e^{2x+3y}}{h}$$

$$= e^{3y}\cdot \lim_{h\to 0}\frac{e^{2(x+h)}-e^{2x}}{h} = e^{2x}\cdot e^{3y}\cdot \lim_{h\to 0}\frac{e^{2h}-1}{h}$$

$$= 2\cdot e^{2x}\cdot e^{3y}\cdot \underbrace{\lim_{h\to 0}\frac{e^{2h}-1}{2h}}_{=1} = 2\cdot e^{2x+3y}$$

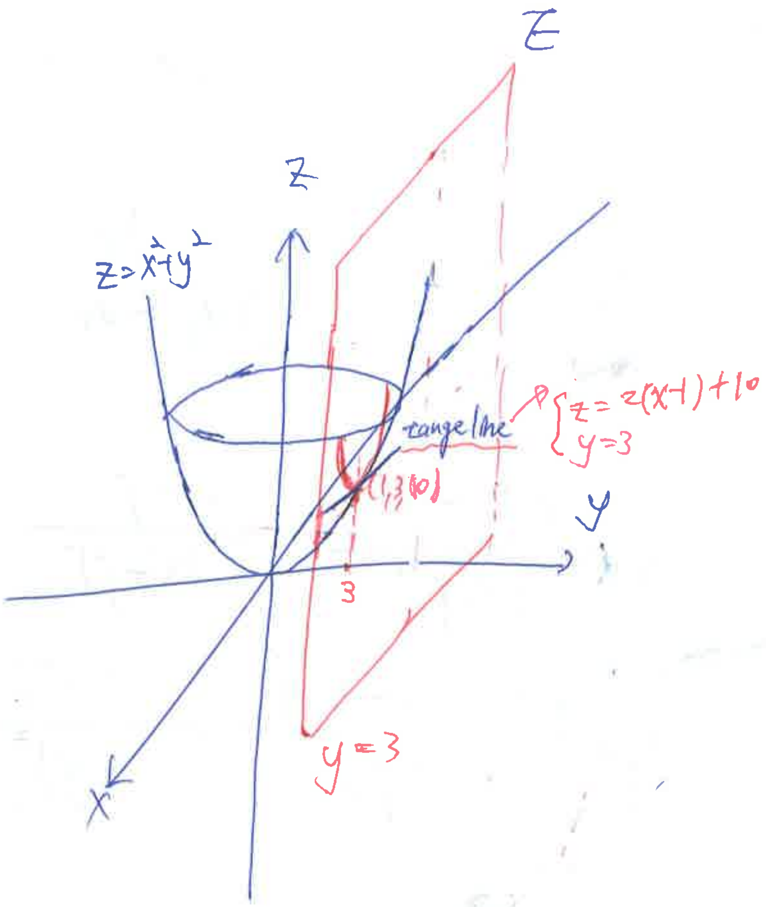
$$\frac{\partial f}{\partial y} = \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h} = \lim_{h\to 0}\frac{e^{2x+3(y+h)}-e^{2x+3y}}{h}$$

$$= \lim_{h\to 0}\frac{e^{2x+3y}\cdot e^{3h}-e^{2x+3y}}{h} = e^{2x+3y}\cdot \lim_{h\to 0}\frac{e^{3h}-1}{h}$$

$$= e^{2x+3y}\cdot 3\cdot \underbrace{\lim_{h\to 0}\frac{e^{3h}-1}{3h}}_{=1} = 3\cdot e^{2x+3y}$$

42. Plane $y=3$; $P(1,3,10)$

$z=x^2+y^2$



Let $f(x,y)=x^2+y^2$

$\Rightarrow f_x=2x \quad , \quad f_y=2y$

$\Rightarrow f_x(1,3)=2 \quad , \quad f_y(1,3)=6$

since tange line lie on plane E,

So, $y=3$

Then, $z=f_x(1,3)(x-1)+10$

$\Rightarrow z=2(x-1)+10$

So, tange line is $\begin{cases} y=3 \\ z=2(x-1)+10 \end{cases}$

[2.

$$u(x,y) = \frac{x}{x^2+y^2} \qquad v(x,y) = \frac{-y}{x^2+y^2}$$

Cauchy Riemann Equation : $\begin{cases} u_x = v_y \\ u_y = -v_x \end{cases}$

$$u_x = \frac{(x^2+y^2) - x\cdot 2x}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2} = v_y$$

$$u_y = \frac{(x^2+y^2)\cdot 0 - x\cdot 2y}{(x^2+y^2)^2} = \frac{-2xy}{(x^2+y^2)^2} = -v_x$$

$$v_x = \frac{(x^2+y^2)\cdot 0 - (-y)\cdot 2x}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2} = -u_y$$

$$v_y = \frac{(x^2+y^2)\cdot(-1) - (-y)\cdot 2y}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2} = u_x$$

⁂

16. $f = f(x,y)$,

$\frac{\partial f}{\partial x} = kf$, k = constant,

Show that $f(x,y) = g(y) \cdot e^{kx}$, $g = g(y)$.

<pf>

By theorem 7.6.1 : If $f'(t) = kf(t)$, $\forall t$, then $\exists\, C$ = constant, such that

$$f(t) = C \cdot e^{kt}, \quad \forall t.$$

Let $f(x,y) = C \cdot e^{kx}$, where C is independent of x.

Since C may depend on y, we write $C = g(y)$.

So, $f(x,y) = g(y) \cdot e^{kx}$ ∎