Hw 7: §13-1 ※ 12, 14, 18

§13-2 ※ 4, 8, 19, 26, 40

§13-3 ※ 6, 12, 16, 30

§13-4 ※ 6, 10, 21, 38, 40.

§13-1

※12.

center: $(1,0,-2)$, radius $=4$

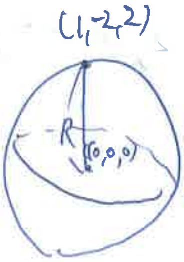
sphere: $(x-1)^2+(y-0)^2+(z-(-2))^2=4^2$

$$\underline{(x-1)^2+y^2+(z+2)^2=16}$$

※14.

center $(0,0,0)$,

radius $=\sqrt{(1-0)^2+(-2-0)^2+(2-0)^2}=\sqrt{1+4+4}=\sqrt{9}=3$



sphere: $(x-0)^2+(y-0)^2+(z-0)^2=3^2$

$$\underline{x^2+y^2+z^2=9}$$

※18. $3x^2+3y^2+3z^2-12x-6z+3=0$

$x^2+y^2+z^2-4x-2z+1=0$

$(x^2-4x+4)+y^2+(z^2-2z+1)=4$

$$\underline{(x-2)^2+(y-0)^2+(z-1)^2=2^2}$$

So, $\underline{\text{center}: (2,0,1)}$ and $\underline{\text{radius}: 2}$

§13-2

※4. $\vec{a} = \vec{PQ}$

$P(-4,0,7) \quad Q(0,3,-1)$

$\vec{a} = \vec{PQ} = Q - P = (0,3,-1) - (-4,0,7) = \underline{(4,\ 3,\ -8)}$

$\|\vec{a}\| = \sqrt{4^2+3^2+(-8)^2} = \sqrt{16+9+64} = \underline{\sqrt{89}}$

※8. $\vec{a} = (1,-2,3)$

$\vec{b} = (3,0,-1)$

$\vec{c} = (-4,2,1)$

$\vec{a} + 3\vec{b} - 2\vec{c} = (1,-2,3) + 3(3,0,-1) - 2(-4,2,1)$

$= (1,-2,3) + (9,0,-3) - (-8,4,2)$

$= \underline{(18,-6,-2)}$

※19. $\vec{a} = i - j + 2k = (1,-1,2)$ $\qquad \vec{d} = -2i + 2j - 4k = (-2,2,-4)$

$\vec{b} = 2i - j + 2k = (2,-1,2)$

$\vec{c} = 3i - 3j + 6k = (3,-3,6)$

since $\vec{d} = -2\vec{a}$, $\vec{c} = 3\vec{a}$, $\frac{-2}{3}\vec{c} = \vec{d}$, so we have

(a) $\underline{\vec{a} \parallel \vec{d},\ \vec{a} \parallel \vec{c},\ \vec{c} \parallel \vec{d}}$

(b) $\underline{\vec{a}, \vec{c}}$ have <u>the same</u> direction.

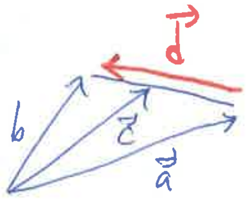
(c) $\underline{\vec{a}, \vec{d}}$ and $\underline{\vec{c}, \vec{d}}$ have <u>opposite</u> direction.

§13-2

26. $\vec{a} = -2i+3j = -2i+3j = (-2,3)$, $\|\vec{a}\| = \sqrt{(-2)^2+3^2} = \sqrt{13}$

unit vector $= \frac{1}{\|\vec{a}\|}\cdot\vec{a} = \frac{1}{\sqrt{13}}(-2,3) = \underline{\left(\frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)}$

40. (i)



since $\vec{a}+\vec{d}=\vec{b}$, $\vec{d}=\vec{b}-\vec{a}$

$\vec{c} = \vec{a} + \frac{1}{2}\vec{d}$

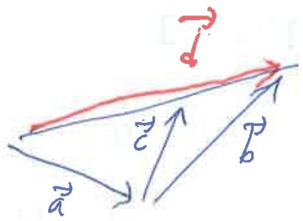
$= \underline{\vec{a} + \frac{1}{2}(\vec{b}-\vec{a})}$

or

$\vec{c} = \vec{b} + (\frac{-1}{2}\vec{d})$

$= \vec{b} - \frac{1}{2}(\vec{b}-\vec{a}) = \underline{\vec{b} + \frac{1}{2}(\vec{a}-\vec{b})}$

(ii)



since $\vec{d} = \vec{a}+\vec{b}$ and $\vec{a} + \vec{c} = \frac{1}{2}\vec{d}$

$\vec{c} = \frac{1}{2}\vec{d} - \vec{a}$

$= \frac{1}{2}(\vec{a}+\vec{b}) - \vec{a}$

$= \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a}$

$= \underline{\frac{1}{2}(\vec{b}-\vec{a})}$

§13-3

*6. $\vec{a} = 2i+3j+k = (2,3,1)$ $\vec{b} = i+4j = (1,4,0)$

$\vec{a}\cdot\vec{b} = 2\times1 + 3\times4 + 1\times0 = \underline{14}$

§13-3

※12. $\vec{a}=\vec{j}+3\vec{k}=(0,1,3)$ $\qquad \|\vec{a}\|=\sqrt{0^2+1^2+3^2}=\sqrt{10}$

$\vec{b}=2\vec{i}-\vec{j}+2\vec{k}=(2,-1,2)$ $\qquad \|\vec{b}\|=\sqrt{2^2+(-1)^2+2^2}=\sqrt{9}=3$

$\vec{c}=3\vec{i}-\vec{k}=(3,0,-1)$ $\qquad \|\vec{c}\|=\sqrt{3^2+0^2+(-1)^2}=\sqrt{10}$

(a) $\vec{a}\cdot\vec{b}=0\times 2+1\times(-1)+3\times 2=\underline{5}$

$\vec{a}\cdot\vec{c}=0\times 3+1\times 0+3\times(-1)=\underline{-3}$

$\vec{b}\cdot\vec{c}=2\times 3+(-1)\times 0+2\times(-1)=\underline{4}$

(b) $\vec{u}_a=\dfrac{\vec{a}}{\|\vec{a}\|}=\dfrac{1}{\sqrt{10}}(0,1,3)=(0,\dfrac{1}{\sqrt{10}},\dfrac{3}{\sqrt{10}})$ $\qquad \vec{u}_c=\dfrac{\vec{c}}{\|\vec{c}\|}=(\dfrac{3}{\sqrt{10}},0,\dfrac{-1}{\sqrt{10}})$

$\vec{u}_b=\dfrac{\vec{b}}{\|\vec{b}\|}=\dfrac{1}{3}(2,-1,2)=(\dfrac{2}{3},\dfrac{-1}{3},\dfrac{2}{3})$

$\cos\theta_1=\vec{u}_a\cdot\vec{u}_b=0\times\dfrac{2}{3}+\dfrac{1}{\sqrt{10}}\times\dfrac{-1}{3}+\dfrac{3}{\sqrt{10}}\times\dfrac{2}{3}=\dfrac{5}{3\sqrt{10}}=\dfrac{5\sqrt{10}}{30}=\underline{\dfrac{\sqrt{10}}{6}}$

$\cos\theta_2=\vec{u}_a\cdot\vec{u}_c=0\times\dfrac{3}{\sqrt{10}}+\dfrac{1}{\sqrt{10}}\times 0+\dfrac{3}{\sqrt{10}}\times\dfrac{-1}{\sqrt{10}}=\underline{\dfrac{-3}{10}}$

$\cos\theta_3=\vec{u}_b\cdot\vec{u}_c=\dfrac{2}{3}\times\dfrac{3}{\sqrt{10}}+\dfrac{-1}{3}\times 0+\dfrac{2}{3}\times\dfrac{-1}{\sqrt{10}}=\dfrac{4}{3\sqrt{10}}=\dfrac{4\sqrt{10}}{30}=\underline{\dfrac{2\sqrt{10}}{15}}$

(c) $\mathrm{comp}_{\vec{b}}\vec{a}=\vec{a}\cdot\vec{u}_b=0\times\dfrac{2}{3}+1\times\dfrac{-1}{3}+3\times\dfrac{2}{3}=\dfrac{-1}{3}+\dfrac{6}{3}=\underline{\dfrac{5}{3}}$

$\mathrm{comp}_{\vec{c}}\vec{a}=\vec{a}\cdot\vec{u}_c=0\times\dfrac{3}{\sqrt{10}}+1\times 0+3\times\dfrac{-1}{\sqrt{10}}=\dfrac{-3}{\sqrt{10}}=\underline{\dfrac{-3\sqrt{10}}{10}}$

(d) $\mathrm{proj}_{\vec{b}}\vec{a}=(\mathrm{comp}_{\vec{b}}\vec{a})\cdot\vec{u}_b=\dfrac{5}{3}(\dfrac{2}{3},\dfrac{-1}{3},\dfrac{2}{3})=\underline{(\dfrac{10}{9},\dfrac{-5}{9},\dfrac{10}{9})}$

$\mathrm{proj}_{\vec{c}}\vec{a}=(\mathrm{comp}_{\vec{c}}\vec{a})\cdot\vec{u}_c=\dfrac{-3\sqrt{10}}{10}(\dfrac{3}{\sqrt{10}},0,\dfrac{-1}{\sqrt{10}})=\underline{(\dfrac{-9}{10},0,\dfrac{3}{10})}$

§13-3

※16. $\vec{a} = 2i - 3j + k = (2, -3, 1)$ $\quad \|\vec{a}\| = \sqrt{14}$

$\vec{b} = -3i + j + 9k = (-3, 1, 9)$ $\quad \|\vec{b}\| = \sqrt{91}$

$\vec{u}_a = \frac{1}{\sqrt{14}}(2, -3, 1) = \left(\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$

$\vec{u}_b = \frac{1}{\sqrt{91}}(-3, 1, 9) = \left(\frac{-3}{\sqrt{91}}, \frac{1}{\sqrt{91}}, \frac{9}{\sqrt{91}}\right)$

$\cos\theta = \vec{u}_a \cdot \vec{u}_b = \frac{-6}{\sqrt{14}\times\sqrt{91}} + \frac{-3}{\sqrt{14}\times\sqrt{91}} + \frac{9}{\sqrt{14}\times\sqrt{91}} = 0 \Rightarrow \underline{\theta = \frac{\pi}{2} = 90^\circ}$

※30. $\vec{a} = xi + 11j - 3k = (x, 11, -3)$ $\quad \vec{a} \perp \vec{b}$

$\vec{b} = 2xi - xj - 5k = (2x, -x, -5)$

$\vec{a} \cdot \vec{b} = 0 \Rightarrow 2x \cdot x + 11 \cdot (-x) + (-3) \cdot (-5) = 0$

$2x^2 - 11x + 15 = 0$

$(2x - 5)(x - 3) = 0$

$\underline{x = \frac{5}{2} \text{ or } x = 3}$

§13-4

※6.

$$i\cdot(j\times k) = i\cdot i = 1$$

※10. $k\cdot(j\times i) = k\cdot(-i\times j) = k\cdot(-k) = -k\cdot k = -1$

※21.

$$\vec{a} = (1,3,-1)$$

$$\vec{b} = (2,0,1)$$

$$\vec{a}\times\vec{b} = \left(\begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} \right)$$

$$= (3, -3, -6)$$

Let $\vec{c} = \vec{a}\times\vec{b} = (3,-3,-6) \Rightarrow \vec{c}\perp\vec{a}$, $\vec{c}\perp\vec{b}$, $\|\vec{c}\| = \sqrt{54} = 3\sqrt{6}$

$$\vec{u}_c = \frac{\vec{c}}{\|\vec{c}\|} = \left(\frac{3}{3\sqrt{6}}, \frac{-3}{3\sqrt{6}}, \frac{-6}{3\sqrt{6}}\right) = \left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$$

$$= \left(\frac{\sqrt{6}}{6}, \frac{-\sqrt{6}}{6}, \frac{-\sqrt{6}}{3}\right)$$

and $\vec{d} = \vec{b}\times\vec{a} = -\vec{a}\times\vec{b} = (-3, 3, 6)$

$\Rightarrow \vec{d}\perp\vec{a}$, $\vec{d}\perp\vec{b}$, $\|\vec{d}\| = 3\sqrt{6}$

$$\vec{u}_d = \frac{\vec{d}}{\|\vec{d}\|} = \left(\frac{-3}{3\sqrt{6}}, \frac{3}{3\sqrt{6}}, \frac{6}{3\sqrt{6}}\right) = \left(\frac{-\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}\right)$$

§ 13-4.

※38. For each vector $\vec{a}$ determine all vector $\vec{b}$ for which $(\vec{a}\times\vec{b})\cdot\vec{b}\neq 0$.

<sol>

since $\vec{a}\times\vec{b} \perp \vec{b}$

$\Rightarrow (\vec{a}\times\vec{b})\cdot\vec{b} = 0.$

So, there are <u>no</u> vectors $\vec{b}$ such that $(\vec{a}\times\vec{b})\cdot\vec{b}\neq 0$.

※

※40. $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular, show that $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{0}$

<pf>

since $\vec{b}\times\vec{c} \perp \vec{b}$ and $\vec{b}\times\vec{c} \perp \vec{c}$

so $\vec{b}\times\vec{c}$ must be parallel to $\vec{a}$

$\Rightarrow \vec{a}\times(\vec{b}\times\vec{c}) = 0$

※