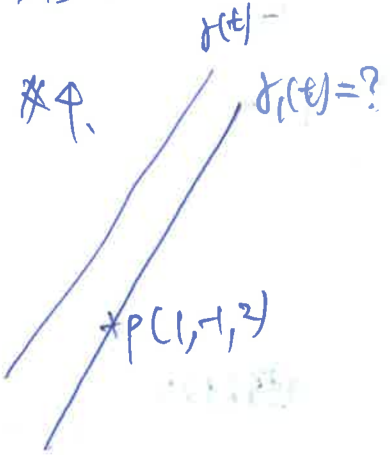
Hw8 §13-5: 4, 10, 14, 20, 22, 30

§13-6: 6, 10, 16, 22, 26, 28, 36, 40

§14-1: 6, 12, 16, 22, 46

§13-5

#4.



Let $r_1(t) = \vec{r}_0 + t\vec{d}$

$\vec{r}_0 = (1,-1,2) = \vec{i} - \vec{j} + 2\vec{k}$

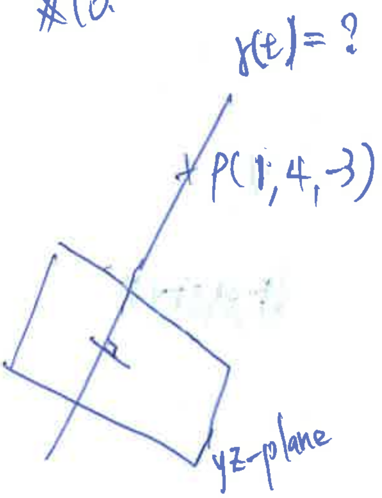
since $r(t) = t(3\vec{i} - \vec{j} + \vec{k})$ and $r(t) \,/\!/\, r_1(t)$

$\Rightarrow \vec{d} = 3\vec{i} - \vec{j} + \vec{k}$

So, $r_1(t) = (\vec{i} - \vec{j} + 2\vec{k}) + t(3\vec{i} - \vec{j} + \vec{k})$

$= (1+3t)\vec{i} + (-1-t)\vec{j} + (2+t)\vec{k}$

#10



$r(t) = \vec{r}_0 + t\vec{d}$

$\vec{r}_0 = P = (1,4,-3) = i + 4\vec{j} - 3\vec{k}$

since the line is perpendicular to the yz-plane

so $\vec{d} = (1,0,0) = \vec{i}$

so $r(t) = i + 4\vec{j} - 3\vec{k} + t\vec{i}$

$= (1+t)\vec{i} + 4\vec{j} - 3\vec{k}$

(2

※14. $l_1: \gamma(t) = (-\vec{i}+2\vec{j}+\vec{k}) + t(\vec{i}-3\vec{j}+2\vec{k}) \Rightarrow \vec{d} = \vec{i}-3\vec{j}+2\vec{k}$

$l_2: R(u) = (2\vec{i}-\vec{j}) + u(-2\vec{i}+6\vec{j}-4\vec{k}) \Rightarrow \vec{D} = -2\vec{i}+6\vec{j}-4\vec{k}$

Since $\vec{D} = -2\vec{d}$. Therefore, the lines are either parallel or coincident.

Since the point $(-1,2,1)$ on l_1 "does not" lie on l_2,

the lines are parallel.

(why?) → suppose $(-1,2,1)$ on l_2

$$-\vec{i}+2\vec{j}+\vec{k} = (2\vec{i}-\vec{j}) + u(-2\vec{i}+6\vec{j}-4\vec{k})$$

$$-\vec{i}+2\vec{j}+\vec{k} = (2-2u)\vec{i} + (-1+6u)\vec{j} + (-4u)\vec{k}$$

$$\begin{cases} 2-2u=-1 \\ -1+6u=2 \\ -4u=1 \end{cases} \Rightarrow u \text{ does not exist.}$$

So, $(-1,2,1)$ does not lie on l_2

※20.

$l_1: x_1(t) = 1+t, \quad y_1(t) = 2t, \quad z_1(t) = 1+3t$

$\gamma(t) = (1+t)\vec{i} + (2t)\vec{j} + (1+3t)\vec{k} = (\vec{i}+\vec{k}) + t(\vec{i}+2\vec{j}+3\vec{k})$

$\Rightarrow \vec{d} = \vec{i}+2\vec{j}+3\vec{k}$

$l_2: x_2(u) = 3u, \quad y_2(u) = 2u, \quad z(u) = 2+u$

$R(u) = (3u)\vec{i} + (2u)\vec{j} + (2+u)\vec{k} = 2\vec{k} + u(3\vec{i}+2\vec{j}+\vec{k})$

$\Rightarrow \vec{D} = 3\vec{i}+2\vec{j}+\vec{k}$

Let $\gamma(t) = R(u)$

$$\Rightarrow (1+t)\vec{i} + (2t)\vec{j} + (1+3t)\vec{k} = (3u)\vec{i} + (2u)\vec{j} + (2+u)\vec{k}$$

$$\Rightarrow \begin{cases} 1+t=3u \\ 2t=2u \\ 1+3t=2+u \end{cases} \Rightarrow u=t=\frac{1}{2}$$

$\therefore \gamma(\tfrac{1}{2}) = \frac{3}{2}\vec{i} + 1\vec{j} + \frac{5}{2}\vec{k}$

So, intersect point $(\frac{3}{2}, 1, \frac{5}{2})$

§13-5

※22.

$l_1: r_1(t) = (\vec{i} - 4\sqrt{3}\vec{j}) + t(\vec{i} + \sqrt{3}\vec{j})$ $\quad \vec{d} = \vec{i} + \sqrt{3}\vec{j} \quad \|\vec{d}\| = 2$

$l_2: r_2(u) = (4\vec{i} + 3\sqrt{3}\vec{j}) + u(\vec{i} - \sqrt{3}\vec{j})$ $\quad \vec{p} = \vec{i} - \sqrt{3}\vec{j} \quad \|\vec{p}\| = 2$

$\Rightarrow \vec{u}_d = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$

$\vec{u}_p = \frac{1}{2}\vec{i} - \frac{\sqrt{3}}{2}\vec{j}$

$\Rightarrow \cos\theta = |\vec{u}_d \cdot \vec{u}_p| = \left|\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \left(\frac{-\sqrt{3}}{2}\right)\right| = \left|\frac{1}{4} - \frac{3}{4}\right| = \left|\frac{-2}{4}\right| = \left|\frac{-1}{2}\right| = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$ (angle)

$r_1(t) = r_2(u)$

$(\vec{i} - 4\sqrt{3}\vec{j}) + t(\vec{i} + \sqrt{3}\vec{j}) = (4\vec{i} + 3\sqrt{3}\vec{j}) + u(\vec{i} - \sqrt{3}\vec{j})$

$(1+t)\vec{i} + (-4\sqrt{3} + \sqrt{3}t)\vec{j} = (4+u)\vec{i} + (3\sqrt{3} - \sqrt{3}u)\vec{j}$

$\begin{cases} 1+t = 4+u \\ -4\sqrt{3} + \sqrt{3}t = 3\sqrt{3} - \sqrt{3}u \end{cases} \Rightarrow \begin{cases} t - u = 3 \\ t + u = 7 \end{cases} \Rightarrow \begin{cases} u = 2 \\ t = 5 \end{cases}$

So, $r_1(5) = 6\vec{i} + \sqrt{3}\vec{j}$

$\therefore$ intersect point $(6, \sqrt{3}, 0)$

30.

$l_1 : r(t) = \vec{r_0} + t\vec{d}$

intersect at right angles.

$l_2 : R(u) = \vec{R_0} + u\vec{D}$

Show that the point of intersection is the origin $\Leftrightarrow$ $r(t) \perp R(u)$, $\forall t \in \mathbb{R}$, $u \in \mathbb{R}$,

<pf> ① suppose the point of intersection is the origin

$\Rightarrow$ there exists number t_0 and u_0 such that $r(t_0) = R(u_0) = 0$

Then $r(t) = r(t) - r(t_0) = (\vec{r_0} + t\vec{d}) - (\vec{r_0} + t_0\vec{d}) = (t - t_0)\vec{d}$

$R(u) = R(u) - R(u_0) = (\vec{R_0} + u\vec{D}) - (\vec{R_0} + u_0\vec{D}) = (u - u_0)\vec{D}$

Since $l_1 \perp l_2$, then $\vec{d} \cdot \vec{D} = 0$

thus, $r(t) \cdot R(u) = (t - t_0)(u - u_0)\,\vec{d} \cdot \vec{D} = 0$. $\forall t, u \in \mathbb{R}$

② suppose $r(t_0) = R(u_0) \neq 0$

Then $r(t_0) \cdot R(u_0) = r(t_0) \cdot r(t_0) = \| r(t_0) \|^2 \neq 0$

and it is therefore not true that $r(t) \cdot R(u) = 0$ for all $t, u \in \mathbb{R}$

that is, if $r(t) \cdot R(u) = 0$ for all $t, u \in \mathbb{R}$

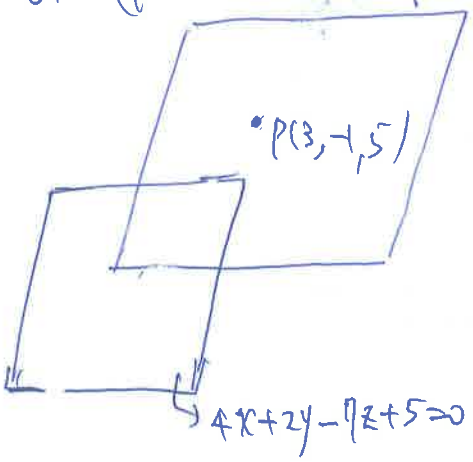
then $r(t_0) = R(u_0) = 0$

$\blacksquare$

5

§ 13-6.

* 6. (parallel)



$\vec{N} = 4\vec{i} + 2\vec{j} - 7\vec{k}$

$P = (3,-1,5)$

$\therefore\ 4(x-3) + 2(y-(-1)) + (-7)(z-5) = 0$

$4x - 12 + 2y + 2 - 7z + 35 = 0$

$\underline{4x + 2y - 7z + 25 = 0}$

* 10.

$2x - 3y + 7z = 0$

$\vec{N} = 2\vec{i} - 3\vec{j} + 7\vec{k}$ $\quad \|\vec{N}\| = \sqrt{4+9+49} = \sqrt{62}$

$\underline{\vec{u}_N = \pm\frac{1}{\sqrt{62}}(2\vec{i} - 3\vec{j} + 7\vec{k})}$

* 16.

$2x - y + 3z = 5$ $\quad \vec{N}_1 = 2\vec{i} - \vec{j} + 3\vec{k}$ $\quad \|\vec{N}_1\| = \sqrt{4+1+9} = \sqrt{14}$

$5x + 5y - z = 1$ $\quad \vec{N}_2 = 5\vec{i} + 5\vec{j} - \vec{k}$ $\quad \|\vec{N}_2\| = \sqrt{25+25+1} = \sqrt{51}$

$\vec{u}_{N_1} = \frac{\pm 1}{\sqrt{14}}(2\vec{i} - \vec{j} + 3\vec{k})$ $\quad \vec{u}_{N_2} = \pm\frac{1}{\sqrt{51}}(5\vec{i} + 5\vec{j} - \vec{k})$

$\Rightarrow \underline{\cos\theta} = |\vec{u}_{N_1} \cdot \vec{u}_{N_2}| = \frac{|(2\vec{i} - \vec{j} + 3\vec{k}) \cdot (5\vec{i} + 5\vec{j} - \vec{k})|}{\sqrt{14} \times \sqrt{51}} = \frac{|2\times5 - 1\times5 - 3\times1|}{\sqrt{14} \times \sqrt{51}}$

$= \underline{\frac{2}{\sqrt{14} \times \sqrt{51}}}$

$\Rightarrow \underline{\theta \doteq 1.5}$

#22.

$\because\ 1\cdot(\vec{j}-\vec{k})+(-1)\cdot(3\vec{i}-\vec{j}+2\vec{k})+1(3\vec{i}-2\vec{j}+3\vec{k})=0$

$\therefore$ coplanar

Let $a(\vec{j}-\vec{k})+b(3\vec{i}-\vec{j}+2\vec{k})+c(3\vec{i}-2\vec{j}+3\vec{k})=0$

$(3b+3c)\vec{i}+(a-b-2c)\vec{j}+(-a+2b+3c)\vec{k}=0$

$$\Rightarrow \begin{cases} 3b+3c=0 \\ a-b-2c=0 \\ -a+2b+3c=0 \end{cases} \Rightarrow \begin{array}{l} b=-c \\ a=c \end{array}$$

※ coplanar $\Rightarrow$ volume $=0$

$\vec{a},\vec{b},\vec{c}$ are coplanar $\Rightarrow (\vec{a}\times\vec{b})\cdot\vec{c}=0$

$\Rightarrow c\cdot(\vec{j}-\vec{k})-c\cdot(3\vec{i}-\vec{j}+2\vec{k})+c(3\vec{i}-2\vec{j}+3\vec{k})=0$

$\Rightarrow c\cdot[(1)\cdot(\vec{j}-\vec{k})-1\cdot(3\vec{i}-\vec{j}+2\vec{k})+1\cdot(3\vec{i}-2\vec{j}+3\vec{k})]=0$

#26.

$p: x+y-2z=0$

$P_0(1,3,4)$

$$d(P_0,P)=\frac{|1+3-2\times4|}{\sqrt{1^2+1^2+(-2)^2}}=\frac{4}{\sqrt{6}}=\frac{4\sqrt{6}}{6}=\frac{2\sqrt{6}}{3}$$

§ 13–6

※ 28. $P_1(1,1,1)$ $P_2(2,-2,-1)$ $P_3(0,2,1)$

$$\overrightarrow{P_1P_2} = (2-1, -2-1, -1-1) = (1,-3,-2)$$

$$\overrightarrow{P_1P_3} = (0-1, 2-1, 1-1) = (-1,1,0)$$

$$\vec{N} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$$

$$= \left(\begin{vmatrix} -3 & -2 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} -2 & 1 \\ 0 & -1 \end{vmatrix}, \begin{vmatrix} 1 & -3 \\ -1 & 1 \end{vmatrix} \right) = (2, 2, -2)$$

$$\Rightarrow 2(x-1) + 2(y-1) - 2(z-1) = 0$$

$$\Rightarrow 2x + 2y - 2z - 2 = 0$$

$$\Rightarrow x + y - z - 1 = 0$$

※ 36. $P_1: x+y+z+1=0$ $\quad$ $P_2: x-y+z+2=0$

$\vec{N_1} = \vec{i} + \vec{j} + \vec{k}$ $\quad$ $\vec{N_2} = \vec{i} - \vec{j} + \vec{k}$

Let $x=0$, we have $\begin{cases} y+z+1=0 \\ -y+z+2=0 \end{cases} \Rightarrow \begin{matrix} y = \frac{1}{2} \\ z = \frac{-3}{2} \end{matrix}$ so, $P(0, \frac{1}{2}, \frac{-3}{2})$: on the line of intersection.

$$\vec{N_1} \times \vec{N_2} = (\vec{i}+\vec{j}+\vec{k}) \times (\vec{i}-\vec{j}+\vec{k})$$

$$= \left(\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \right) = (2, 0, -2) = 2\vec{i} - 2\vec{k}$$
$$= 2(\vec{i} - \vec{k})$$

$\Rightarrow$ $\vec{d} = \vec{i} - \vec{k}$, $\vec{r_0} = \frac{1}{2}\vec{j} - \frac{3}{2}\vec{k}$

Thus, Line: $r(t) = \vec{r_0} + t\vec{d} = \frac{1}{2}\vec{j} - \frac{3}{2}\vec{k} + t(\vec{i} - \vec{k})$

$$= t\vec{i} + \frac{1}{2}\vec{j} + (-t - \frac{3}{2})\vec{k}$$

#40. Show that two nonparallel lines $\gamma(t)=\vec{r}_0+t\vec{d}$ and $R(u)=\vec{R}_0+u\vec{D}$ intersect $\Leftrightarrow$ the vector $\vec{r}_0-\vec{R}_0$, $\vec{d}$, and $\vec{D}$ are coplanar.

18

<pf> ($\Rightarrow$)

suppose two nonparallel lines intersect

$\Rightarrow$ there exists two numbers t_0, u_0 such that $\gamma(t_0)=R(u_0)$

$\Rightarrow \vec{r}_0+t_0\vec{d}=\vec{R}_0+u_0\vec{D}$

$\Rightarrow \vec{r}_0-\vec{R}_0+t_0\vec{d}-u_0\vec{D}=0$

$\Rightarrow 1\cdot(\vec{r}_0-\vec{R}_0)+t_0\vec{d}-u_0\vec{D}=0$, so, $\vec{r}_0-\vec{R}_0$, $\vec{d}$, $\vec{D}$ are coplanar.

($\Leftarrow$)

suppose $\vec{r}_0-\vec{R}_0$, $\vec{d}$, $\vec{D}$ are coplanar, there exists three numbers α, β, γ, such that $\alpha(\vec{r}_0-\vec{R}_0)+\beta\vec{d}+\gamma\vec{D}=0$

we may assume $\alpha\neq 0$.

(if $\alpha=0$, we have $\beta\vec{d}+\gamma\vec{D}=0$, then $\vec{d}$ and $\vec{D}$ are parallel (contradiction))

$\Rightarrow \vec{r}_0-\vec{R}_0+\frac{\beta}{\alpha}\vec{d}+\frac{\gamma}{\alpha}\vec{D}=0$

$\Rightarrow \vec{r}_0+\frac{\beta}{\alpha}\vec{d}=\vec{R}_0-\frac{\gamma}{\alpha}\vec{D}$

$\Rightarrow \gamma\left(\frac{\beta}{\alpha}\right)=R\left(\frac{-\gamma}{\alpha}\right)$

$\Rightarrow$ two nonparallel lines intersect.

$\blacksquare$

§14-1

※6.

$f(t) = e^t\vec{i} + te^t\vec{j} + t^2e^t\vec{k}$

$f'(t) = e^t\vec{i} + (e^t + te^t)\vec{j} + (2te^t + t^2e^t)\vec{k}$

$= e^t\left[\vec{i} + (1+t)\vec{j} + (2t+t^2)\vec{k}\right]$

※12

$f(t) = \sqrt{t}\,\vec{i} + t\sqrt{t}\,\vec{j} + \ln t\,\vec{k}$

$f'(t) = \frac{1}{2\sqrt{t}}\vec{i} + \left(\sqrt{t} + \frac{t}{2\sqrt{t}}\right)\vec{j} + \frac{1}{t}\vec{k} = \frac{1}{2\sqrt{t}}\vec{i} + \frac{3}{2}\sqrt{t}\,\vec{j} + \frac{1}{t}\vec{k}$

$f''(t) = \frac{-1}{4t\sqrt{t}}\vec{i} + \frac{3}{4\sqrt{t}}\vec{j} + \left(\frac{-1}{t^2}\right)\vec{k}$

※16

$\int_0^{\pi} \sin t\,\vec{i} + \cos t\,\vec{j} + t\,\vec{k}\; dt$

$= \int_0^{\pi} \sin t\, dt\,\vec{i} + \int_0^{\pi} \cos t\, dt\,\vec{j} + \int_0^{\pi} t\, dt\,\vec{k}$

$= -\cos t\,\Big|_0^{\pi}\vec{i} + \sin t\,\Big|_0^{\pi}\vec{j} + \frac{1}{2}t^2\Big|_0^{\pi}\vec{k}$

$= 2\vec{i} + 0\vec{j} + \frac{1}{2}\pi^2\vec{k}$

#22.

$$f(t) = 3(t^2-1)\vec{i} + \cos t\,\vec{j} + \frac{t}{|t|}\vec{k}$$

$$\lim_{t\to 0} f(t) = \lim_{t\to 0} 3(t^2-1)\vec{i} + \lim_{t\to 0}\cos t\,\vec{j} + \lim_{t\to 0}\frac{t}{|t|}\vec{k}$$

$\because \lim_{t\to 0}\frac{t}{|t|}$ does not exist

$\therefore \lim_{t\to 0} f(t)$ does not exist

#46.

$$f'(t) = t\vec{i} + t(1+t^2)^{-\frac{1}{2}}\vec{j} + te^t\vec{k} \qquad f(0) = \vec{i} + 2\vec{j} + 3\vec{k}$$

$$f(t) = \int t\,dt\,\vec{i} + \int t(1+t^2)^{-\frac{1}{2}}dt\,\vec{j} + \int te^t dt\,\vec{k}$$

$$\int t\,dt = \frac{1}{2}t^2 + C_1 \qquad \int t(1+t^2)^{-\frac{1}{2}}dt = \int \frac{1}{2}u^{-\frac{1}{2}}du = u^{\frac{1}{2}} = (1+t^2)^{\frac{1}{2}} + C_2$$

$u = 1+t^2$

$du = 2t\,dt$

$$\int te^t dt = \int t\,d(e^t) = te^t - \int e^t dt = te^t - e^t + C_3$$

$$\therefore f(t) = \left(\frac{1}{2}t^2 + C_1\right)\vec{i} + \left((1+t^2)^{\frac{1}{2}} + C_2\right)\vec{j} + \left(te^t - e^t + C_3\right)\vec{k}$$

$$f(0) = C_1\vec{i} + (1+C_2)\vec{j} + (C_3-1)\vec{k} = \vec{i} + 2\vec{j} + 3\vec{k}$$

$$C_1 = 1 \quad C_2 = 1 \quad C_3 = 4$$

$$\therefore f(t) = \left(\frac{1}{2}t^2+1\right)\vec{i} + \left((1+t^2)^{\frac{1}{2}}+1\right)\vec{j} + \left(te^t - e^t + 4\right)\vec{j}$$