Names and Student IDs: _____

Homework 3 Calculus 2

1. Salas § 8.6, Problem 29
2. (a) Given $\lambda \in (0,1)$, prove that $h : \mathbb{R}^+ \to \mathbb{R}$ defined by $h(t) = t^\lambda - \lambda t$ has absolute maximum at $t = 1$. What is the maximum value?

   (b) With part (a), do Rudin Chapter 6, Problem 10 (a),(b). Here, change all α to x (ie. we deal with usual Riemann integrals here).
3. Salas § 8.6, Problem 43.
4. Continue from Problem 2, prove

   (a) For any $f, g \in \mathcal{R}([a,b])$, we have

$$\left| \int_a^b fg dx \right| \leq \left(\int_a^b |f|^p dx \right)^{\frac{1}{p}} \left(\int_a^b |g|^q dx \right)^{\frac{1}{q}}.$$

   (b) The equality hold if and only if $f = cg$ for some $c \neq 0$.
5. Prove that

$$\binom{n}{k} = \left[(n+1) \int_0^1 x^k (1-x)^{n-k} \, dx. \right]^{-1}$$

6. Rudin Chapter 6 Problem 15 (Hint: Use Problem 2, 4). For the strict inequality $>$, just prove $\geq$.
7. Salas § 8.5: 2, 6, 15, 22, 28.
8. Salas § 8.6: 2, 12.
9. Salas § 8.7: 5, 9, 12.
10. Salas § 11.7: 4, 18, 25, 30, 32, 39, 54, 58, 66, 67.
11. (5 points extra credit) For Problem 6, prove that the inequality is actually strict.