Names and Student IDs: ____________________

Homework 4 Calculus 2

1. Rudin Chapter 6, Problem 9.
2. Prove that
   (a) Prove that
   $$\Gamma(x+1) = x\Gamma(x)$$
   for all $x \in (0, \infty)$.
   (b) Prove that
   $$\Gamma(x) = \frac{2^{x-1}}{\sqrt{\pi}}\Gamma\left(\frac{x}{2}\right)\Gamma\left(\frac{x+1}{2}\right).$$
   for all $x \in (0, \infty)$.
3. Prove that
   (a)
   $$\Gamma(n) = 2\int_0^\infty u^{2n-1}e^{-u^2}\,du.$$
   (b)
   $$\Gamma(n) = \int_0^1 \left(\log\frac{1}{x}\right)^{n-1}\,dx.$$
4. Use Stirling's formula to show that
   $$\lim_{x\to\infty}\frac{\Gamma(x+c)}{x^c\Gamma(x)} = 1,$$
   for all $x \in \mathbb{R}$.
5. Rudin Chapter 6, Problem 16.
6. Use Theorem 8.20 in Rudin and Problem 4 to show that
   $$\lim_{n\to\infty}\sqrt{n}\int_{-1}^1 (1-x^2)^n\,dx = \sqrt{\pi}.$$
7. Salas § 13.3: 10, 13, 30, 41, 47, 52.
8. Salas § 13.4: 20, 41, 42, 46, 47.
9. Salas § 13.5: 9, 11, 16, 28, 31, 34, 43.
10. Salas § 13.6: 7, 18, 22, 26, 30, 36, 41, 44.