Note 1.1 Limit: Definitions and Basic Concepts

1 Introduction

Let's now get to work and define what "small" or "close" mean in calculus. We really mean "infinitely small" and "infinitely close", which we may not really obtain. But, some observations tell us that there seem to be something out there that we will obtain if we really repeat procedures infinitely many times. That something is called the limit. Let's define them with intuition and rigorous mathematical languages.

2 Intuition of Limit

We have probably heard the example of cake eating (or other food). You eat half of it the first day, then $\frac{1}{4}$, $\frac{1}{8}$, .. on the coming days. We all agree that we *eventually* eat the whole cake. What does this mean precisely?

Let's be more mathematical:

Here is a function that does not act like previous ones:

Let's define the limit of a function verbally from these observations.

3 Precise Definition

Here is the real, rigorous mathematical definition of limit for your reference. We will not deal with it (or at least not entirely) in this course.

Definition 3.1. Let $f : A \to B$ and $c \in \mathbb{R}$. We say that f has limit L at c, written

$$\lim_{x \to c} f(x) = L,$$

if:

For every $\epsilon > 0$, there exists $\delta > 0$ so that for every x with $0 < |x - c| < \delta$, we have $|f(x) - L| < \epsilon$.

Some comments on the definition:

What about $c = \infty$?