Note 4.1 - Related Rates and Differentials

1   Introduction

We apply chain rules/implicit differentiations and linear approximations to some practical problems.

2   Related Rates

In many situations, there are quantities, related by equation(s), depend upon each other:
$$F(x, y) = 0.$$

It can be mathematically unclear to tell which one is independent variable or dependent variable (i.e. the function). They can also be functions of another variable (the *parameter*) as well:

$$F(x(t), y(t)) = 0.$$

In any case, we are interested to see how a certain variable varies with respect to one of the other variables or parameters. This is done by implicit differentiation.

3 Differentials

We have discussed how differentiations are interpreted as linear approximations.

Now we use them to approximate some numerical values of functions:

The tangent lines are also quick tools to estimate the change of function value:

$$\Delta f := f(x + \Delta x) - f(x)$$

These are often used to estimate the errors of some quantities: