# Note 3.4 - Derivatives of Exponential and Logarithmic Functions 

## 1 Introduction

Let's fill in the last piece of puzzle: the derivatives of $e^{x}$ and $\ln x$. The results will strongly reflect the facts that exponential functions are the standard functions to describe quantities whose rate of change depend on the quantities themselves.

## 2 The Derivative of $e^{x}$

To differentiate $e^{x}$, we first derive the following limit:

$$
\lim _{h \rightarrow 0} \frac{e^{h}-1}{h}=1
$$

Then, the derivative of $e^{x}$ follows easily from the power rules.

We see that the rate of change of $e^{x}$ is itself!

## 3 The Derivative of $\ln x$

The derivative of $\ln x$ can be derived easily with our available tools:

## 4 Arbitrary Bases

With our definitions of $a^{x}$ and $\log _{a} x$ in Note 1 , their derivatives are now easy to derive.

5 Examples

